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Is multivariate probit really useless in analysing off-farm labour participation of farm couples? A note

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ABSTRACT

In a recent paper, Benjamin and Kimhi (2006) argued that using multivariate probit for analysing off-farm participation of farm couples suffers from a theoretical inconsistency. This note tries to clarify this issue, and illustrates the joint participation rules based on reservation wages of household members. The reservation wages of each household member is contingent on the participation status of the other member. Hence, participation rules have to keep into account the joint participation probabilities of the couple.

Keywords: off-farm labour participation, farm household, multivariate probit
JEL: J22, J43, Q12

1. Introduction

In a recent paper, Benjamin and Kimhi (2006) argued that using multivariate probit for analysing off- and on-farm participation of farm couples suffers from a theoretical inconsistency. As a consequence, a whole stream of literature on joint participation decision-making of farm couples and households, starting from the seminal paper by Huffman and Lange (1989), appears to be flawed. The goal of this paper is to try and clarify this issue and, more precisely, to ascertain under which qualification the approach of using the comparison between the reservation and the market wage as the decision rule for participation is valid when the decision-making is joint within the household.

2. The theoretical model

The theoretical model underlying the analysis of on- and off-farm labour participation of farm couples (and, possibly, other household members) is the standard farm household model (Singh et al., 1986; Huffman, 2001). The farm household is assumed to maximise utility over household consumption and leisure of family members, under income and time constraint. The income constraint comprise both farm income and off-farm wages. Non-negativity constraints are added to allow for non-participation in on- and off-farm work. The model (for simplicity, we refer here to a farm couple without other family members) is as follows:

$$
\begin{aligned}
&Max\,U = U\left(C, L^{g}; H^{g}, Z_{H}\right) \quad g = o, s\\
&S.t.:\\
&T^{g} = F^{g} + M^{g} + L^{g},\\
&L^{g} > 0,\\
&F^{g}, M^{g} \geq 0,\\
&Q = f\left(F^{g}, X; H^{g}, Z_{F}\right)\\
&PQ + \sum W^{g} M^{g} + V = C + RX\\
&W^{g} = W^{g}(H^{g}, Z_{M})
\end{aligned}
\qquad (1)
$$

where:

C: family consumption

L: leisure

g = o, s refers to operator and spouse

H: vector of personal variables, e.g. age and education

Z_H: vector of characteristics of the household, e.g. number of children

T: total available time

F, M time spent working on the farm and off the farm, respectively

Q: quantity of the good produced by the farm

X: vector of hired inputs

Z_F: vector of farm characteristics

P: price of the good produced by the farm

W: off-farm market wage

V: value of capital income

R: vector of prices of purchased inputs

Z_M: vector of characteristics of the off-farm market

The Kuhn-Tucker maximisation conditions yield the following on-farm and off-farm participation conditions:

$$\delta PQ / \delta F^{g} \leq \mu / \lambda \qquad (2)$$

$$W^{g} \leq \mu / \lambda \qquad (3)$$

where μ and λ are the marginal utilities of leisure and income, respectively (for simplicity, we use here family labour input rather than the effective labour income used by Benjamin and Kimhi, a fixed external wage, and we assume that hired labour is not perfectly substitutable to family labour). The first condition states that participation in farm work (an internal solution) occurs if the marginal value product of farm work is equal to the leisure-income marginal rate of substitution. The second states that participation in off-farm work occurs if the wage is equal to the leisure-income MRS.

If an interior solution occurs for all choices, then the participation equations (2) and (3) and the constraints in (1) can be solved for the endogenous variables as functions of all exogenous variables. Reduced-form participation equations are obtained by comparing the reservation wage (the MRS evaluated at zero hours) with the market wage and the marginal value product of farm labour with the market wage. If the difference is negative, the relevant participation occurs. In empirical applications, both the reservation and the market wage are expressed as functions of their relevant explanatory variables; by adding normal stochastic terms, probit techniques are used to estimate the participation equations.

3. Reservation wages and participation rules

The problem Benjamin and Kimhi point out is that, when the participation choice is not made by a single but by a couple, the reservation wage of each member is a function, among other exogenous variables, also of the wage of the other member, but only if he/she does participate. They show that in case of a couple, for each member two reservation wages exist, one when the other member participates in off-farm work, the other when the other member does participate.

Up to here, Benjamin and Kimhi's (2006) argument is perfectly correct, as well as their claim that the multinomial logit approach does not suffer any theoretical problem of this kind (see also Benjamin et al., 1996). Nevertheless, a closer look should be given at the participation problem, to assess whether the multivariate probit approach is actually theoretically inconsistent.

Call w^*_{o1} and w^*_{o2} the operator's reservation wage when the spouse does and does not participate in off-farm work, respectively; w^*_{s1} and w^*_{s2} the spouse's reservation wage when the operator does and does not participate in off-farm work, repectively; w_o and w_s the operator's and the spouse's market wage, respectively. Then, calling O_1 and O_0 the events that the operator does and does not participate, and S_1 and S_0 the events that the spouse does and does not participate, respectively, the probability that the operator works off-farm is:

$$Prob(O_1)= Prob(w^*_{o1}< w_o| S_1) + Prob(w^*_{o2}< w_o| S_0) \qquad (4)$$

Of course, also the spouse's participation choices are conditional on the operator's participation choices. This directly stems from the simultaneous nature of the decision process assumed by the model. There are overall 16 combinations of relationships between the reservation wages and the market wages of the operator and the spouse, that dictate the outcomes of participation of both members. They are shown, along with the resulting outcomes, in Table 1.

Table 1: Combinations of reservation wages and participation regimes

	$w^*_{s1}< w_s$, $w^*_{s2}< w_s$	$w^*_{s1}< w_s$, $w^*_{s2}\geq w_s$	$w^*_{s1}\geq w_s$, $w^*_{s2}< w_s$	$w^*_{s1}\geq w_s$, $w^*_{s2}\geq w_s$
$w^*_{o1}< w_o$, $w^*_{o2}< w_o$	Case 1: O_1, S_1	Case 2: O_1, S_1	Case 3: O_1, S_0	Case 4: O_1, S_0
$w^*_{o1}< w_o$, $w^*_{o2}\geq w_o$	Case 5: O_1, S_1	Case 6: O_1, S_1	Case 7: O_0, S_0	Case 8: O_0, S_0
$w^*_{o1}\geq w_o$, $w^*_{o2}< w_o$	Case 9: O_0, S_1	Case 10: O_0, S_0	Case 11: O_0, S_0	Case 12: O_1, S_0
$w^*_{o1}\geq w_o$, $w^*_{o2}\geq w_o$	Case 13: O_0, S_1	Case 14: O_0, S_0	Case 15: O_0, S_1	Case 16: O_0, S_0

The cases can be interpreted as follows:

a) the operator participates regardless of the spouse's situation: cases 1, 2, 3, and 4. Both when the spouse's reservation wage is higher and lower than her market wage, the operator's reservation wage is lower than his market wage;

b) the operator does not participate regardless of the spouse's situation: cases 13, 14, 15, and 16. This is because both his reservation wage is in any case lower than his market wage;

c) the operator participates because the spouse always participates, regardless of the operator's situation: case 5. In fact, the operator would participate only if the spouse would also participate;

d) the operator participates because the spouse never participates, regardless of the operator's situation: case 12. In this case, the operator would participate only if the spouse would not participate;

e) the operator does not participate because the spouse always participates, regardless of the operator's situation: case 9. If the spouse would not participate, the operator would participate;

f) the operator does not participate because the spouse never participates, regardless of the operator's situation: case 8. It is easy to see that the operator would participate if the spouse would also do;

g) the operator does not participate because this choice would be incompatible with the spouse's: cases 6, 7, 10, 11. For instance, in case 6, the operator would participate if the spouse would also do, but her reservation wage is higher than her market wage if the operator participates. Also, the operator's reservation wage when the spouse does not participate is higher than his market wage.

Of course, some combinations can be void. Indeed, it seems unlikely that a household member's reservation wage be lower when the other member does not participate than when he/she participates, because the higher income provided by the other member participation decreases the marginal utility of income.

Similar considerations apply to the spouse's situation. One should in any case avoid interpreting the choices as one member deciding after observing the other member's choice, since choices are, by the model's assumption, simultaneous. In this sense, the critique of Benjamin and Kimhi (2006) to the endogenous switching model of Kimhi (1999) as imposing a uni-directional conditioning of the participation equations is correct. It also follows from the above considerations that the bivariate probit approach to deal with the off-farm labour participation choices of farm couples presented in the seminal paper by Huffman and Lange (1989) needs a qualification. Since the reservation wage is not unique, but depends on the participation choice of the other member, and since also the explanatory variables affecting the reservation wage are different depending on the participation choice of the other member, the parameters of a probit participation equation cannot be interpreted as the difference between the effect of the relevant variable on the reservation and on the market wage.

To see this, it is convenient to recall Huffman and Lange's (1989) econometric model of off-farm labour participation:

$$M^o = w_o\alpha_{o11} + w_s\alpha_{o12} + Z\alpha_o + \mu_o \text{ if } w_o \geq w^*_o \text{ and } w_s \geq w^*_s \quad (5)$$

$$M^o = w_o\alpha_{o11}^* + Z\alpha_o^* + \mu_o^* \text{ if } w_o \geq w^*_o \text{ and } w_s < w^*_s \quad (6)$$

$$M^o = 0 \text{ otherwise;} \quad (7)$$

$$M^s = w_s\alpha_{s21} + w_o\alpha^o{}_{21} + Z\alpha_s + \mu_s \text{ if } w_o \geq w^*_o \text{ and } w_s \geq w^*_s \quad (8)$$

$$M^s = w_s\alpha_{s21}^* + Z\alpha_s^* + \mu_s^* \text{ if } w_o < w^*_o \text{ and } w_s \geq w^*_s \quad (9)$$

$$M^s = 0 \text{ otherwise;} \quad (10)$$

$$w_i = X_i\beta_i,\ i = o,s \quad (11)$$

where M^o and M^s are dichotomous variables indicating participation of the operator and of the spouse, respectively, Z is a vector of nonwage variables that are exogenous to farm household consumption, production, and labour allocation choices, X_i is a vector of personal and labour market characteristics, and α_{ijk}, α_i, and β_i are unknown parameters.

Equations (5) and (8) apply when both members participate, and equations (6) and (9) when the other member does not participate. A bivariate probit model is used for estimation, due to the likely correlation among the disturbance terms.

Benjamin and Kimhi's (2006) argument is that, when considering the two different reservation wages, the sum of the probabilities of the four possible participation combinations (O_1, S_1; O_1, S_0; O_0, S_1; O_0, S_0) exceeds one. The implied probabilities are indicated by them as follows, in our notation:

$$
\begin{aligned}
&Prob(O_1, S_1) = Prob(w^*_{o1} < w_o, w^*_{s1} < w_s) \\
&Prob(O_1, S_0) = Prob(w^*_{o2} < w_o, w^*_{s1} < w_s) \qquad (12) \\
&Prob(O_0, S_1) = Prob(w^*_{o1} < w_o, w^*_{s2} < w_s) \\
&Prob(O_0, S_0) = Prob(w^*_{o2} < w_o, w^*_{s2} < w_s)
\end{aligned}
$$

They conclude that the sum of the four probabilities exceeds one by the amount $Prob(w^*_{o1} > w_o > w^*_{o2}, w^*_{s1} > w_s > w^*_{s2})$, which is positive if leisure is a normal good. But if the conditions for participation are properly taken into account (see Table 1), the relevant probabilities are the following:

$Prob(O_1, S_1) = Prob[(w^*_{s1} < w_s, w^*_{s2} < w_s, w^*_{o1} < w_o, w^*_{o2} < w_o) \cup (w^*_{s1} < w_s, w^*_{s2} \geq w_s, w^*_{o1} < w_o, w^*_{o2} < w_o) \cup (w^*_{s1} < w_s, w^*_{s2} < w_s, w^*_{o1} < w_o, w^*_{o2} \geq w_o) \cup (w^*_{s1} < w_s, w^*_{s2} \geq w_s, w^*_{o1} < w_o, w^*_{o2} \geq w_o)]$ (Cases 1, 2, 5, 6)

$Prob(O_1, S_0) = Prob[(w^*_{s1} \geq w_s, w^*_{s2} < w_s, w^*_{o1} < w_o, w^*_{o2} < w_o) \cup (w^*_{s1} \geq w_s, w^*_{s2} \geq w_s, w^*_{o1} < w_o, w^*_{o2} < w_o)) \cup (w^*_{s1} \geq w_s, w^*_{s2} \geq w_s, w^*_{o1} \geq w_o, w^*_{o2} < w_o)]$ (Cases 3, 4, 12)

$Prob(O_0, S_1) = Prob[(w^*_{s1} < w_s, w^*_{s2} < w_s, w^*_{o1} \geq w_o, w^*_{o2} < w_o) \cup (w^*_{s1} < w_s, w^*_{s2} < w_s, w^*_{o1} \geq w_o, w^*_{o2} \geq w_o) \cup (w^*_{s1} \geq w_s, w^*_{s2} < w_s, w^*_{o1} \geq w_o, w^*_{o2} \geq w_o)]$ (Cases 9, 13, 15)

$Prob(O_0, S_0) = Prob[(w^*_{s1} \geq w_s, w^*_{s2} < w_s, w^*_{o1} < w_o, w^*_{o2} \geq w_o) \cup (w^*_{s1} \geq w_s, w^*_{s2} \geq w_s, w^*_{o1} < w_o, w^*_{o2} \geq w_o) \cup (w^*_{s1} < w_s, w^*_{s2} \geq w_s, w^*_{o1} \geq w_o, w^*_{o2} < w_o) \cup (w^*_{s1} \geq w_s, w^*_{s2} < w_s, w^*_{o1} \geq w_o, w^*_{o2} < w_o) \cup$

$(w^*_{s1} < w_s,\ w^*_{s2} < w_s,\ w^*_{o1} \geq w_o,\ w^*_{o2} \geq w_o) \cup (w^*_{s1} \geq w_s,\ w^*_{s2} \geq w_s,\ w^*_{s1} < w_s,\ w^*_{s2} < w_s)]$ (Cases 7, 8, 10, 11, 14, 16) (13)

It is immediately evident that the four probabilities sum to one. What Benjamin and Kimhi (2006) fail to consider is that the participation rules are contingent on the joint decision-making, since the reservation wages too depend on the joint decision-making.

From the above table it is also easy to see that the probability for the operator of having an off-farm job is the sum of the probabilities of cases 1, 2, 3, 4, 5, and 12.

The corresponding probabilities in terms of the market and reservation wages depend on whether $w^*_{o1} > w^*_{o2}$ and $w^*_{s1} > w^*_{s2}$. There are then four hypotheses that can be retained.

Hp A: $w^*_{o1} > w^*_{o2}$, $w^*_{s1} > w^*_{s2}$.

The overall probability of Cases 1, 2, 3, and 4 is $Prob(O_1) = Prob(w_o > w^*_{o1})$, since for the operator to participate, his market wage is to be greater than both w^*_{o1} and w^*_{o2}, and $w^*_{o1} > w^*_{o2}$.

The probability of Case 5 is nil, since the market wage cannot be at the same time greater than w^*_{o1} and lower than w^*_{o2}, since, by Hp A, $w^*_{o1} > w^*_{o2}$.

The probability of Case 12 is $Prob(O_1) = Prob(w^*_{o1} < w_o < w^*_{o2},\ w_s < w^*_{s2})$.

Overall, following Hp A, the probability of the operator's participation is:

$$Prob(O_1) = Prob(w_o > w^*_{o1}), + Prob(w^*_{o1} < w_o < w^*_{o2},\ w_s < w^*_{s2})$$

Hp B: $w^*_{o1} > w^*_{o2}$, $w^*_{s1} < w^*_{s2}$.

The overall probability of Cases 1, 2, 3, and 4 is again $Prob(O_1) = Prob(w_o > w^*_{o1})$.

The probability of Case 5 is again nil.

The probability of Case 12 is now $Prob(O_1)=Prob(w^*_{o1}<w_o<w^*_{o2}, w_s<w^*_{s1})$.

Overall, following Hp B, the probability of the operator's participation is:

$Prob(O_1) = Prob(w_o > w^*_{o1}), + Prob(w^*_{o1}<w_o<w^*_{o2}, w_s<w^*_{s1})$.

Hp C: $w^*_{o1}< w^*_{o2}, w^*_{s1}< w^*_{s2}$.

The overall probability of Cases 1, 2, 3, and 4 is $Prob(O_1) = Prob(w_o > w^*_{o2})$, since for the operator to participate, his market wage is to be greater than both $w^*_{o1}$and w^*_{o2}, and $w^*_{o1}< w^*_{o2}$.

The probability of Case 5 is $Prob(O_1)=Prob(w^*_{o1}<w_o<w^*_{o2}, w_s>w^*_{s2})$.

The probability of Case 12 is nil, since the market wage cannot be at the same time greater than w^*_{o2} and lower than w^*_{o1}, since $w^*_{o1}< w^*_{o2}$

Overall, following Hp C, the probability of the operator's participation is:

$Prob(O_1) = Prob(w_o > w^*_{o2}), + Prob(w^*_{o1}<w_o<w^*_{o2}, w_s>w^*_{s2})$.

Hp D: $w^*_{o1}< w^*_{o2}, w^*_{s1}< w^*_{s2}$.

The overall probability of Cases 1, 2, 3, and 4 is again $Prob(O_1) = Prob(w_o > w^*_{o2})$.

The probability of Case 5 is $Prob(O_1)=Prob(w^*_{o1}<w_o<w^*_{o2}, w_s>w^*_{s1})$.

The probability of Case 12 is again nil.

Overall, following Hp D, the probability of the operator's participation is:

$$\text{Prob}(O_1) = \text{Prob}(w_o > w^*_{o2}), + \text{Prob}(w^*_{o1}<w_o<w^*_{o2}, w_s>w^*_{s1}).$$

The situations corresponding to the four Hypotheses are represented in Panels A), B), C), and D) in the Appendix, in which the shadowed areas depict the probability of operator's participation on the operator's-spouse's wages plane.

4. Conclusions

This note has explored the implications of the argument put forward by Benjamin and Kimhi (2006) that the bivariate approach to modeling off-farm labour participation of farm couples suffers of a theoretical inconsistency. According to their argument, one cannot model participation as the probability of a positive difference between the market and the reservation wage, because two reservation wages exist, contingent to the other member's participation status.

We have showed that indeed the argument is correct. The approach of modeling off-farm labour participation as the probability of a positive difference between the market and the reservation wage can nevertheless be retained, provided that the appropriate joint participation probability is considered. In fact, it has been showed that the probability of participation for a member is the sum of the univariate probability of his/her market wage being larger than the reservation wage and a portion of the bivariate joint probability for both spouses. Operationally, though, a major difficulty with writing a likelihood function seems to be that different cases are possible depending on whether the reservation wage when the other member participates is greater than the reservation wage when the other does not participates) or vice versa. The first hypothesis seems more realistic, considering that when the other member participates, income is greater, and hence it the reservation wage should be higher, but this should be more formally shown. This issue is left to further research.

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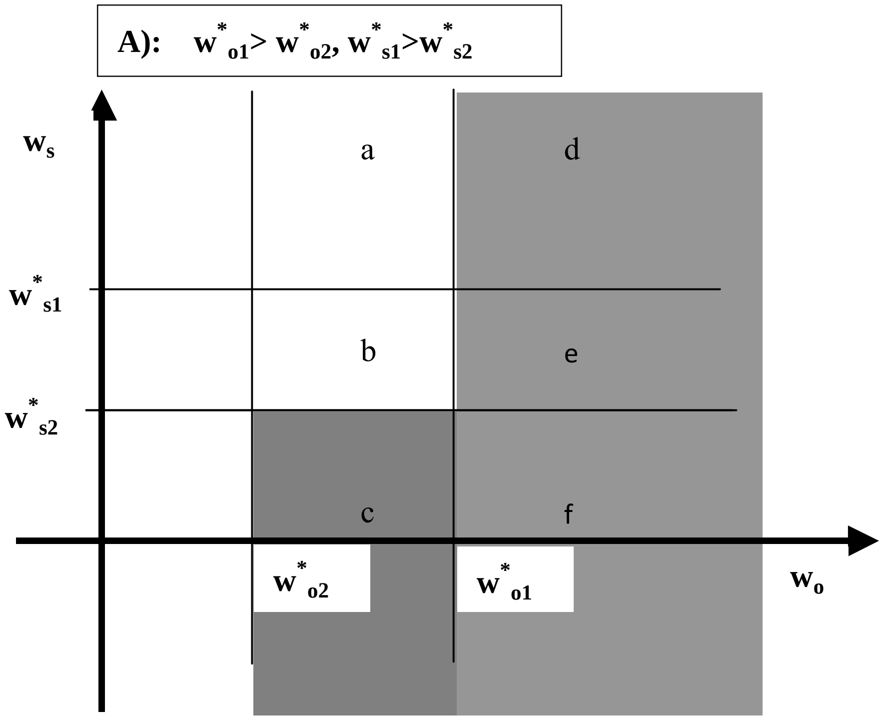
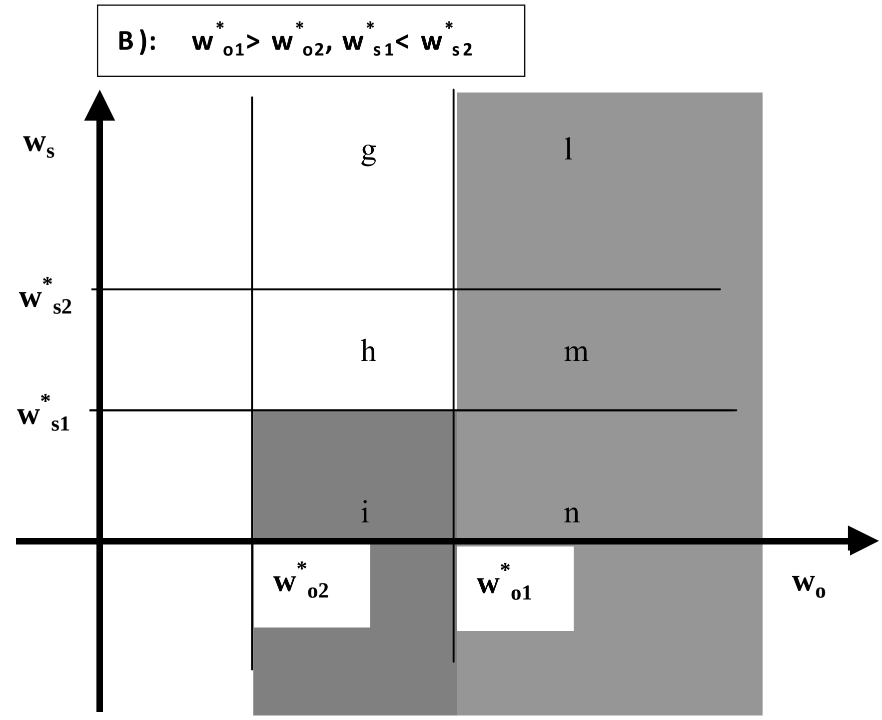
Appendix

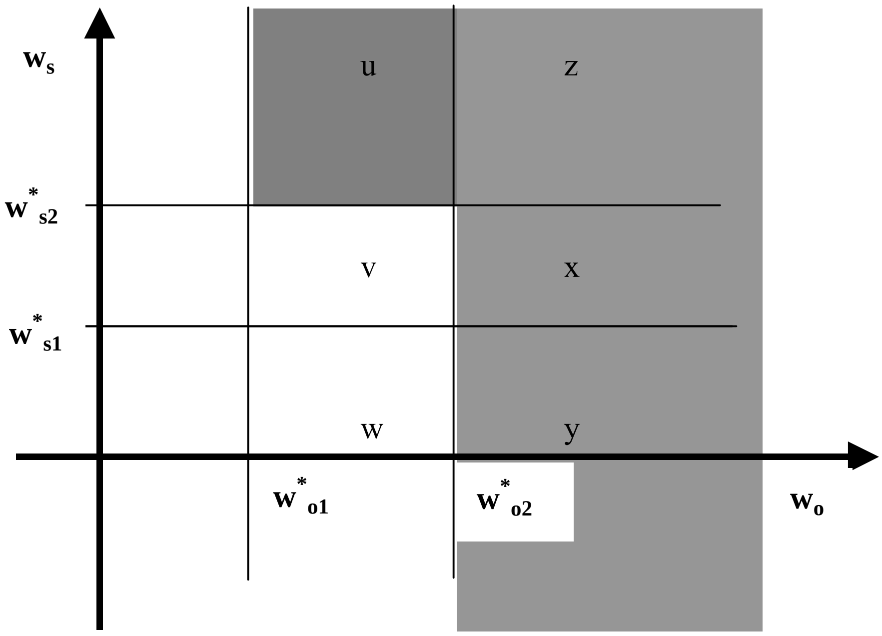
The probability of off-farm work participation for the operator ($Pr(O_1)$ is the union of the probabilities of Cases 1, 2, 3, 4, 5, and 12, as presented in Table 1.

The conditions defining $Pr(O_1)$ are different according to whether w^*_{o1} is larger or smaller than w^*_{o2} and to whether w^*_{s1} is larger or smaller than w^*_{s2}. The conditions according to the four combinations resulting from the relative positions of w^*_{o1}, w^*_{o2}, and w^*_{s1}, w^*_{s2} are represented in Panels A), B), C), and D), depicting on the plane w_o- w_s the probability areas of an off-farm labour participation of the operator.


A): w*o1> w*o2, w*s1>w*s2
ws
w*s1
w*s2
a
b
c
d
e
f
w*o2
w*o1
wo


B): w*o1> w*o2, w*s1< w*s2
ws
w*s2
w*s1
g
h
i
l
m
n
w*o2
w*o1
wo

C): $w^*_{o1} < w^*_{o2}, w^*_{s1} < w^*_{s2}$



D): $w^*_{o1} < w^*_{o2}, w^*_{s1} > w^*_{s2}$