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Optimal savings and heallth spending over the life cycle

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Optimal savings and heallth spending over the life cycle

Tamara Fioroni*

Abstract

This paper investigates the relationship between saving and health spending in a two-period overlapping generations economy. Individuals work in the first period of life and live in retirement in old age. Health spending is an activity that increases the quality of life and longevity. Empirical evidence shows that both health spending and saving behave as luxury goods but their behavior differs markedly according to the level of per capita GDP. The share of saving on GDP has a concave shape with respect to per capita GDP, whereas the share of health spending on GDP increases more than proportionally with respect to per capita GDP. Their ratio is nonlinear with respect to income, i.e. first increasing and then decreasing. This ratio, in the proposed model, is equal to the ratio between the elasticity of the utility function with respect to saving and the elasticity of the utility function with respect to health.

Keywords: Intertemporal Choice, Health Spending, Adult Mortality, Saving
JEL Classification: D91, I12, E21

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1 Introduction

Through the last two centuries, economic development has gradually contributed to the increase in the human life span. In 1840 life expectancy at birth was 40 years in England, 44 years in Denmark and 45 years in Sweden (Livi-Bacci, 2001). According to recent life tables, in 2007 life expectancy at birth in the United Kingdom, Denmark and Sweden is 79, 78 and 81 years respectively. In most developed countries, life expectancy at birth is around 80 years (CIA, The World Factbook 2007). Developing countries have also shown a rapid increase in life expectancy, though this trend has now levelled off. Indeed, since 1980 in several low-income countries the HIV/AIDS epidemic has reversed the positive trend in life expectancy (Becker et al., 2005; Cutler et al., 2006), and many developing countries showed a lower life expectancy at birth in 1980 than in 1960. Mortality rates are much higher in poor countries, with a difference in life expectancy between rich and poor countries of about 30 years (Cutler et al., 2006). This brings the world's average down to 66 years (CIA, The World Factbook 2007).

The increase in life expectancy has significant implications for various aspects of society. In the literature, Bloom et al. (2003), Kageyama (2003), and Zhang et al. (2003), for example, show that increases in life expectancy lead to higher savings rates. This is because agents, in their working age, increase their saving to finance higher consumption needs in old age (Modigliani and Brumberg, 1980). Blackburn and Cipriani (2002) show that higher longevity promotes human capital accumulation. Although this literature has gained important insights into the relationship between life expectancy and economic growth, life expectancy is assumed exogenous or dependent on human capital. Thus the explicit effect of health spending on life expectancy and, through this channel, the relationship between total resources devoted to health care and total resources devoted to nonhealth consumption.

The recent health economics literature, being based on earlier theoretical contributions on the demand for health, focuses on the direct effect of health investment on life expectancy, on the concept of the value of life and the willingness-to-pay criterion to reduce mortality risk (Murphy and Topel, 2006; Ehrlich and Yin, 2005). Such studies emphasize the importance of both quality and quantity of life for overall economic welfare. The central idea is that living is a generally enjoyable activity for which consumers should be willing to sacrifice other pleasures. On the other hand, agents demand health since it increases the time available for market and non-market activities. Indeed, a rise in the stock of health reduces the amount of time lost on these activities, and the monetary value of this reduction is an index of the return to the investment in health (Grossman, 1972).

According to this approach, the paper of Jones and Hall (2006), focusing on the optimal choice between length of life and consumption, shows that health behaves as a superior good. That is, as income rises the marginal utility of consumption falls more quickly than the marginal utility of health spending. Jones and Hall (2006)'s paper is closely related to the seminal empirical paper of Newhouse (1977) who estimates that income elasticity of medical care expenditure is greater than one.

Our research departs from this literature by stressing the effect of health investment on life expectancy and quality of life. This framework allows us to investigate the agent's decisions on the allocation of total resources between saving and health investment. It is assumed that agents have an initial stock of health that depreciates over time and can be increased by investing in health services. People value health spending for two reasons. First, it increases their life expectancy. Second, it is a source of utility, that is it allows them to enjoy better life in any instant of time.

Empirical analysis shows that both health spending and saving, i.e. consumption when old, appear to be luxury goods but their behavior differs greatly according to the level of per capita GDP. The share of saving on GDP has a concave shape with respect to per capita GDP whereas the share of health spending on GDP increases more than proportionally with respect to per capita GDP. The ratio of saving to health investment is nonlinear with respect to per capita GDP: it is first increasing and then decreasing.

Our model can replicate this empirical evidence by assuming a complementarity between health and saving. As income increases, the marginal utility of consumption decreases whereas the marginal utility of health increases. Hence, as people become richer and older they have a higher propensity to invest in health care.

The structure of the paper is as follows. Section 2 presents empirical analysis. Section 3 shows the model. Finally, some concluding remarks are made in section 4.

2 Empirical evidence

The data used in the analysis are taken from World Development Indicators (World Bank, 2006). They cover the period 1960-2005 and 208 countries. In Figure 2 we present a recent version of the Preston curve (1975), that is the international relationship between adult survival rate and per capita GDP in purchasing power parity[1]. With respect to Preston (1975) who uses the data on life expectancy we use the data on the survival rate that are less sensitive to child mortality. This is because we are interested

[1]The survival rate is the difference between 1 and the adult mortality rate. The adult mortality rate is defined by the World Bank as the probability of dying between the ages of 15 and 60, that is, the probability of a 15-year-old dying before reaching age 60, if subject to current age specific mortality rates between ages 15 and 60.

in the adult's health investment decisions to improve his\her probability of surviving to old age[2]. We estimate the Preston curve using a cross-country nonparametric regression (year 2002, 158 countries).

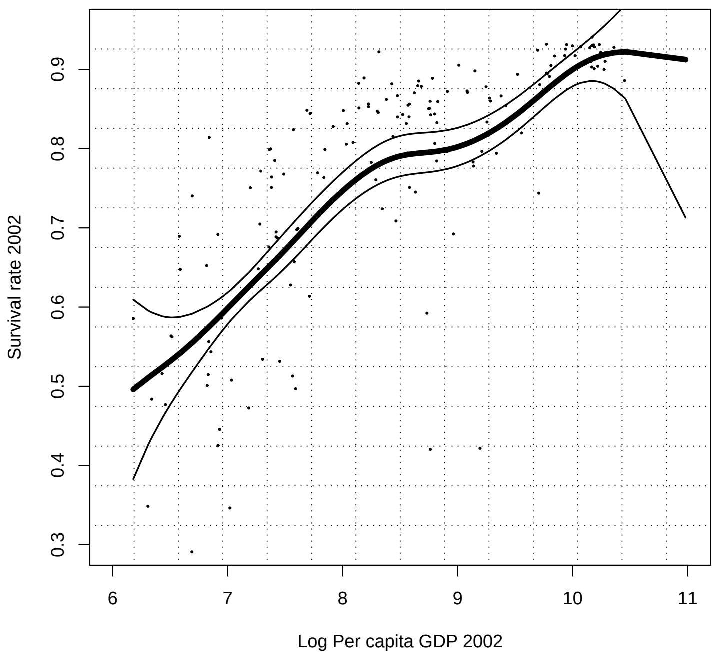

Figure 1: The Preston Curve: Survival Rate versus GDP Per Capita. Nonparametric kernel smoother (bandwidth = 0.45), year 2002, n = 158. Source: World Development Indicators CD-ROM, World Bank (2006)

We prefer to perform nonparametric regression since it allows us to investigate the relationship between the dependent variable and one or more explanatory variables, without making any a priori explicit or implicit assumption about the shape of such relationship. In Figure 2 the confidence interval clearly identifies a positive relationship between survival rate and per capita GDP. In particular, the confidence interval is an indication of the degree of variability in the estimate but it cannot be used to draw firm conclusions about the shape of the curve in particular regions [3]. To assess the shape of the curve we carry out a test which compares nonparametric regression with a simple

[2]However, if we use the data on life expectancy the results are very similar.

[3]The confidence interval describes the level of variability present in the estimate without attempting to adjust for the inevitable presence of bias. The width of the confidence interval is determined by an estimate of the standard error (Bowman and Azzalini, 1997; Hardle et al., 2004). However the confidence interval cannot be used to assess the shape of the curve in particular regions. This is not only because of the presence of bias, but also due to the pointwise nature of the bands (Bowman and Azzalini, 1997).

linear regression. This test indicates that the relationship between survival rate and per capita GDP can be represented by a linear model, i.e. the significance test for the nonparametric regression shows a $p - value = 0.119$. However, in Figure 2 we can see that the relationship is not clearly linear: indeed, in low income countries, increases in per capita GDP are strongly associated with increases in life expectancy; as income per head rises the relationship flattens out. This path reflects the influence of a country's own level of income on mortality through such factors as nutrition, education, leisure and health spending. Concerning health spending, Figure 2 shows the direct relationship between survival rate and per capita health investment in 2002 for 155 countries. Per capita health investment includes both public and private expenditures on health. It covers the provision of health services (preventive and curative), family planning activities, nutrition activities, and emergency aid designated for health but does not include provision of water and sanitation (World Bank, 2006). The relationship between survival rate and per capita health expenditure is clearly positive and can be represented by a linear model ($p - value = 0.618$). However, as in the Preston curve, Figure 2 shows that countries with low levels of health expenditure tend to gain more in life expectancy than countries starting with high level of health spending.

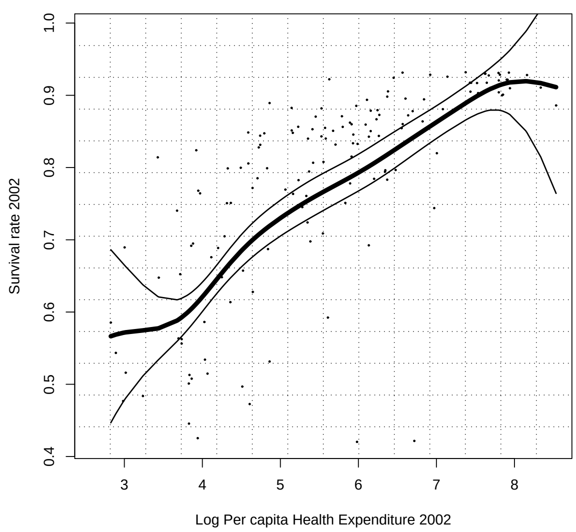

Figure 2: Survival rate versus per capita Health Expenditure. Nonparametric kernel smoother (bandwidth = 0.53), year 2002, n = 155. Source: World Development Indicators CD-ROM, World Bank (2006)

In figures 3 and 4 we examine the path of health expenditure and saving with respect to income[4]. The aim is to analyze the behavior of health spending with respect to

[4]We pool all observations in the period 1997-2002 for 147 countries.

different levels of income and the relationship between saving and health investment, that is between agents preferences for consumption in old age and health spending devoted to increase longevity and quality of life.

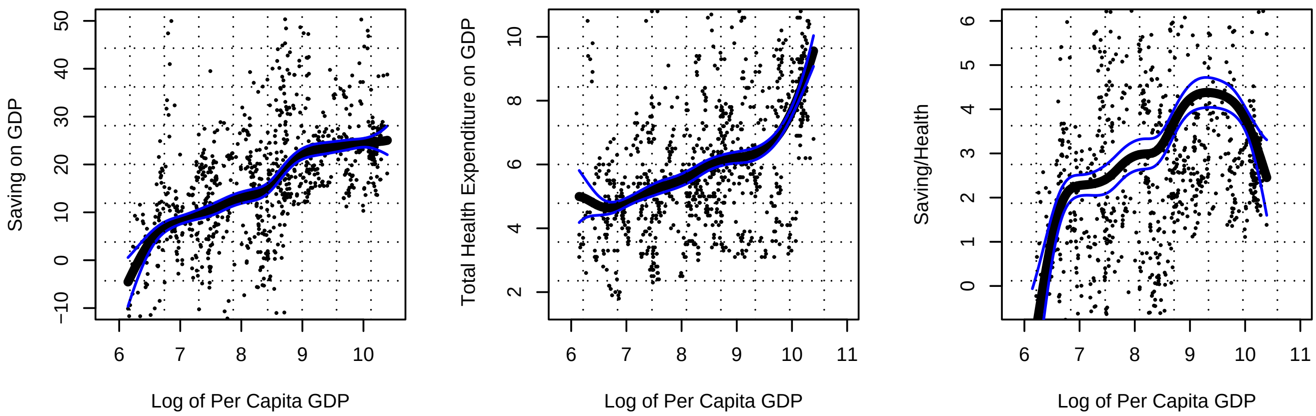

Figure 3: Saving and Health versus GDP Per Capita. Nonparametric kernel smoother (bandwidth = 0.31), years from 1997 to 2002, n = 863. Source: World Development Indicators CD-ROM, World Bank (2006)

Figure 3 shows that both health expenditure on GDP and saving on GDP present a luxury goods behavior. However, their path is strongly different according to different levels of per capita GDP. The share of saving on GDP has a concave shape with respect to per capita GDP. By contrast, health spending on GDP increases more than proportionally with respect to per capita GDP. The comparison between nonparametric regression and a simple linear model yields that the linear model can be rejected for both saving and health spending ($p - value = 0$).

The path of the ratio between saving and health expenditure is clearly non linear: it is first increasing and then decreasing ($p - value = 0$). This suggests that investment in health increases faster than saving when a country is sufficiently developed. The intuition is that as income rises, the saturation occurs faster in saving than in health spending.

Figure 4 compares the path of the saving share and the health share. When the log of per capita GDP is very low (6 and 7) the saving share is below the health investment. This result can be explained by the fact that health investment covers part of public expenditure as emergency aid. When income rises, health on GDP grows faster than the saving share.

It is possible to find different explanations for this behavior of health spending. One

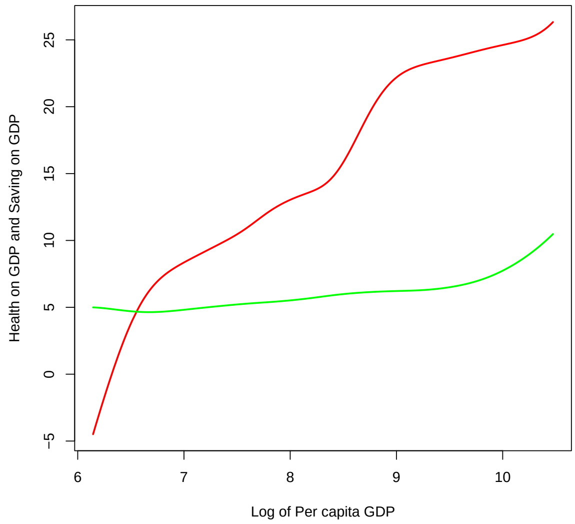

Figure 4: Saving Share and Health Share. Years from 1997 to 2002. Source: World Development Indicators CD-ROM, World Bank (2006)

may be the progressiveness of the tax schedule since the average tax rate increases with income. Others explanations are based on individual's preferences. The idea is that as income grows agent's preferences extend not only to the amount of goods consumed but also to the length of life which allows an additional period of utility to be enjoyed (Jones, 2004; Jones and Hall, 2006). In other words, as people become richer, they decide to increase their consumption of health services to extend their life expectancy. In the next section we propose a model based on this explanation.

3 The Model

Consider an overlapping generations economy where members of generation t live for two periods: adulthood and old age. Each agent, at time t, inherits an initial level of health h_t which depreciates with age and can be augmented by the investment. If health at the end of the first period falls below a minimum level $\underline{h}$ agents do not survive to the second period of life.

Parents are non-altruistic and if they do not survive to old age, their saving is passed onto their offspring as unintended bequest. Hence, in the first period of life each agents has an endowed level of wealth as unintended bequest, $b_t \geq 0$, and work receiving a constant wage equal to $\overline{w}$. Total resources of agents, i.e. $y_t = \overline{w} + b_t$, are allocated between current consumption, health expenditure and saving devoted to financing old age consumption. Thus, in the first period, the budget constraint of the representative agent is:

$$c_t = y_t - m_t - s_t, \tag{1}$$

where m_t is the health investment[5] and s_t is the saving.

In old age, agents live in retirement and consume their savings entirely. Hence the budget constraint is given as follows:

$$c_{t+1} = s_t R, \tag{2}$$

where R is the constant interest rate in the period $t+1$.

We suppose that health investment performs two distinct services (Ehrlich and Chuma, 1990; Murphy and Topel, 2006): first, it raise the quality of life, that is it augments the amount of healthy time available in any instant of life; second, it raises the length of life. In particular, death takes place when the health stock of agents falls below a minimum level, i.e. $\underline{h}$ (Grossman, 1972).

Health level is assumed a concave function of health investment, i.e. m_t:

$$h_{t+1} = h(m_t),$$

$$h'(m_t) > 0, \; h''(m_t) < 0.$$

In particular, we specify health level as follows :

$$h_{t+1} = \left[h_t(1-\rho) + m_t^{\delta}\right]^{\frac{1}{\delta}}, \tag{3}$$

where h_t is the level of health that agents inherit a the beginning of period t, $\rho < 1$ is the depreciation rate of h_t, and δ determines the concavity of h_{t+1} (see Appendix A).

[5]We suppose perfect substitutability between public health expenditure and private health spending. This implies that a higher proportion of government expenditure devoted to health services reduces private health spending. In particular, the health investment m_t, is assumed to be the sum of private health investment, m_t^{PRI}, and public health investment, m_t^{PUB}. The latter is financed by a proportional tax on income, that is $m_t^{PUB} = \tau y_t$. Hence, the agent's budget constraint in the first period is given by:

$$c_t = (1-\tau)\,y_t - m_t^{PRI} - s_t,$$

where substituting $m_t^{PUB} = \tau y_t$ we obtain:

$$c_t = y_t - s_t - \left(m_t^{PRI} + m_t^{PUB}\right),$$

where $m_t^{PRI} + m_t^{PUB} = m_t$ in equation (1).

The idea is that if agents pay high taxes they receive high quality public health services and therefore decide to devote a low proportion of income to private health expenditure. Otherwise, when the public health sector does not provide any health services, health spending is only private.

Assumption 1. The parameter δ satisfies the following condition:

$$\delta < 1.$$

Preferences of a member of generation t are defined over consumption c_t and health stock h_t in adulthood and consumption c_{t+1} and health stock h_{t+1} in old age:

$$U_t = u(c_t, h_t) + \beta u(c_{t+1}, h(m_t)), \tag{4}$$

where $u(c_t, h_t)$ is the utility in the first period, $0 < \beta < 1$ is the psychological discount factor. When $h(m_t) \geq \underline{h}$, agents survive to the second period of life and they enjoy a utility which depends on consumption and health level, when $h(m_t) < \underline{h}$ agents do not survive to the second period of life (Grossman, 2004).

3.1 Optimal saving and health spending

Proposition 1 characterizes the optimal condition for saving and health spending:

Proposition 1 *The optimal allocation of total resources implies that the ratio of saving to health investment is:*

$$\frac{s_t}{m_t} = \frac{\varepsilon_{u_s}}{\varepsilon_{u_m}}, \tag{5}$$

where ε_{u_s} is the elasticity of the instantaneous utility function with respect to saving and ε_{u_m} is the elasticity of the utility function with respect to health investment[6]*.*

Proof. Given the budget constraints in equations (1) and (2), the first order conditions with respect to s_t and m_t are:

$$\frac{u_s(c_t, h_t)}{u_s(c_{t+1}, h(m_t))} = \beta R, \tag{6}$$

and:

$$\frac{u_m(c_t, h_t)}{u_m(c_{t+1}, h(m_t))} = \beta h'(m_t), \tag{7}$$

where u_i is the derivative of the utility function with respect to the i-th argument. The substitution of equation (7) in equation (6) yields the ratio between saving and health investment. ■

Equation (6) is the usual condition that requires the marginal rate of substitution between current and future consumption to be equal to the expected return on saving.

[6]The elasticity of the utility function with respect to saving can be defined as the percentage change in the utility function in response to a given change in saving.

The elasticity of the utility function with respect to health investment can be defined as the percentage change in the utility function in response to a given change in health spending.

Equation (7) captures the trade-off between the marginal cost and marginal benefit of health care spending. By investing in health care, agents give up current consumption to increase their quality and quantity of life to the second period.

According to Proposition 1 the response of the ratio between saving and health spending to variations in the level of income depends on the behavior of the elasticities in equation (5). Empirical evidence (Figures 3 and 4) shows that both saving and health investment rise with income but, when income is high, health spending on GDP grows faster than the saving on GDP. The intuition is that when income becomes higher than a certain threshold, consumption rises but agents prefer to devote an increasing share of resources to health care.

3.2 Preferences

Jones and Hall (2006) to explain the luxury good behavior of health spending, choose to add a constant term to the standard utility function with constant elasticity of substitution (C.E.S). Using this specification in our model we obtain intractable results.

Following Murphy and Topel (2006); Ehrlich and Chuma (1990) we suppose that health level and consumption of other goods are complements, that is $u_{c,h} \geq 0$. For example, if health level declines at the old age, then consumption will also fall despite one's incentive to defer consumption. Hence, we specify the lifetime utility in equation (4) as follows:

$$U_t = (c_t)^{1-\phi} (h_t - \underline{h})^{\phi} + \beta (c_{t+1})^{1-\phi} (h(m_t) - \underline{h})^{\phi}, \tag{8}$$

where the utility from health level in the first period is positive, i.e. $h_t > \underline{h}$, and $\phi < 1$ ensures that $u_{c,h} \geq 0$. From equation (3) the level of health spending below which agents do not survive to old age is given by:

$$\hat{m} = \left[(\underline{h})^{\delta} - h_t (1-\rho) \right]^{1/\delta}, \tag{9}$$

which is positive, that is:

$$h_t < \frac{(\underline{h})^{\delta}}{1-\rho}. \tag{10}$$

Given equation (10) and that $h_t > \underline{h}$ it follows:

$$\underline{h} < \left(\frac{1}{1-\rho} \right)^{\frac{1}{1-\delta}}.$$

From equations (6) and (7) we get the optimal saving as a function of health investment:

$$s_t = \left(\frac{1-\phi}{\phi} \right) \left(\frac{h(m_t) - \underline{h}}{h'(m_t)} \right). \tag{11}$$

From equations (11) and (6) we obtain the following implicit relation between health investment and income, that is:

$$\left(\frac{1-\phi}{\phi}\right)\left(\frac{\mu+h\left(m_t\right)-\underline{h}}{h'\left(m_t\right)}\right)-y_t+m_t=0, \tag{12}$$

where $\mu = R(h_t - \underline{h})/(\beta R)^{1/\phi}$.
We are interested in analyzing the behavior of saving and health investment according to different levels of per capita income. The aim is to show that, as income rises the marginal utility of consumption falls quickly whereas the marginal utility of health spending increases. The intuition is that as agents become richer and older is more valuable an additional year of life than a third car or more clothing or a another house. As result, the optimal composition of total spending shifts toward health spending.

Optimal share of health, i.e. m_t/y_t, increases with respect to income if the following assumption holds.

Assumption 2. The parameters satisfy the following condition:

$$\mu > \underline{h}.$$

The following propositions define the properties of health share and saving share.

Proposition 2 *Under Assumptions 1 and 2 optimal health share satisfies the following properties:*

- $\lim\limits_{m\to\hat{m}} \frac{m_t}{y_t} = \frac{1}{1+\gamma} > 0,$
- $\lim\limits_{m\to\infty} \frac{m_t}{y_t} = \phi,$
- $\frac{\partial(m_t/y_t)}{\partial y_t} > 0.$

Proof. The technical part of this proposition is proved in Appendixes B and C. ■

Proposition 3 *Under Assumptions 1 and 2 optimal saving share in income satisfies the following properties:*

- $\lim\limits_{m\to\hat{m}} \frac{s_t}{y_t} = 0,$
- $\lim\limits_{m\to\infty} \frac{s_t}{y_t} = 1-\phi,$
- $\frac{\partial(s_t/y_t)}{\partial y_t} > 0.$

Proof. See Appendix D ■

Propositions 2 and 3 imply that both saving and health investment behave as superior goods with an income elasticity above one (see appendix C and D). Thus, both health spending and saving increase faster than income.

This result, is confirmed by many empirical contributions which analyze the path of health spending with respect to income. For example, Newhouse (1977) regresses per capita health spending on per capita GDP for 13 developed countries in the years from 1968 to 1972. He finds the income elasticity of health care expenditures exceeds one, ranging from 1.15 to 1.31, and concludes that medical care is a luxury good. Blomqvist and Carter (1997) estimate, for example, that the income elasticity of health spending, for OECD countries in the period 1960 to 1991, is significantly above one.

We now turn to an analysis of the ratio between saving and health investment which is can be obtained simply from equation (11) above:

$$\frac{s_t}{m_t} = \left(\frac{1-\phi}{\phi}\right)\left(\frac{h(m_t) - \underline{h}}{m_t h'(m_t)}\right). \tag{13}$$

where $\varepsilon_{u_c} = 1 - \phi$ and $\varepsilon_{u_m} = \phi m_t h'(m_t) / (h(m_t) - \underline{h})$. Hence, the path of the ratio between saving and health spending depends crucially on the movement of ε_{u_m} with respect to income. The crux of our argument is that the consumption elasticity is constant whereas health elasticity depends on the level of health spending and consequently on the level of income. As a long as Assumption 1 is satisfied, the model shows that for low level of income, ε_{u_m} decreases with respect to income, and for high level of income it increases with respect to income. This implies that for poor agents the percentage change in the utility function in response to a given change in health spending is positive but decreasing. In contrast, for rich agents the percentage change in the utility function in response to a given change in health spending is increasing. Therefore, poor agents have a higher propensity to consume while rich agents have a higher propensity to invest in health care.

Proposition 4 *Under Assumptions 1 and 2 the ratio between saving and health spending satisfies the following properties (see figure 5):*

- $\lim_{m \to \hat{m}} \frac{s_t}{m_t} = 0$,
- $\lim_{m \to \infty} \frac{s_t}{m_t} = \frac{1-\phi}{\phi}$,
- $\frac{\partial (s_t/m_t)}{\partial y_t} > 0$ *for* $y_t < \overline{y}$, *while* $\frac{\partial (s_t/m_t)}{\partial y_t} < 0$ *for* $y_t > \overline{y}$.

Proof. The technical part of this proposition is proved in Appendix E ■

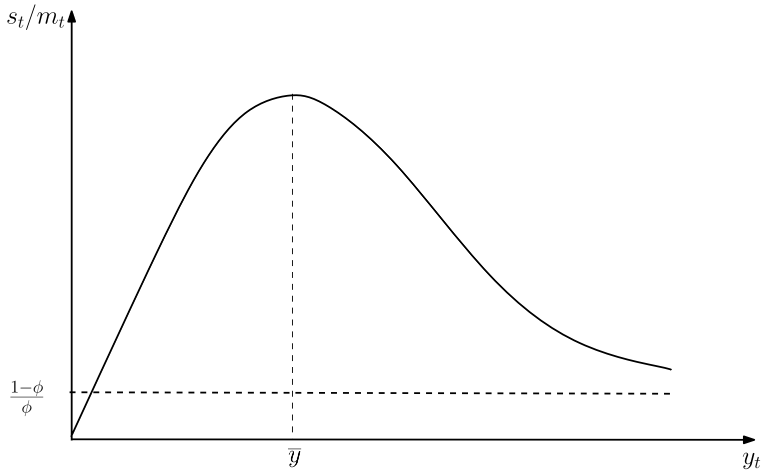

Figure 5: Ratio between Saving and Health Expenditure versus Income.

Figure (5) illustrates the path of the ratio between saving and health investment for different income levels. It shows that for income level $y_t < \overline{y}$, saving grows more quickly than health investment, hence the ratio s_t/m_t is increasing as income increases. In the opposite, when $y_t > \overline{y}$ the ratio s_t/m_t is decreasing as income increases.

4 Conclusions

This paper analyzes agent's decisions on the allocation of total resources between health investment and saving. Empirical evidence shows that when income is low agents devote more income to saving to ensure consumption in old age. As income rises the saving continues to rise but health spending increases more quickly. The intuition for such results is that as income grows people become saturated in non-health consumption and choose to spend more income to obtain a better health status and additional years of life. This empirical evidence is supported with a theoretical model in which health and consumption of other goods are supposed complements. The proposed model implies that while the marginal utility of consumption falls rapidly the marginal utility of health spending does not run into the same kind of diminishing returns. As income rises, agents give more value to their health level and their length of life than to their consumption of other goods.

Appendix

A Health Production Function

Given equation (3) we get that:

$$\frac{\partial h_{t+1}}{\partial m_t} = {m_t}^{\delta-1}\left[h_t\left(1-\rho\right)+m_t^{\delta}\right]^{\frac{1}{\delta}-1} > 0, \tag{14}$$

and:

$$\frac{\partial^2 h_{t+1}}{\partial m_t^2} = -m_t^{\delta-2}h_t\left(1-\rho\right)\left(1-\delta\right)\left[h_t\left(1-\rho\right)+m_t^{\delta}\right]^{\frac{1}{\delta}-2} < 0, \tag{15}$$

if:

$$\delta < 1$$

B Existence of a solution

Equation (12) has at least a real solution if there exists a value m_t so that:

$$\left(\frac{1-\phi}{\phi}\right)\left(\frac{\mu+h\left(m_t\right)-\underline{h}}{h'\left(m_t\right)}\right)+m_t = y_t \tag{16}$$

We define the function on the left side of equation (16) as $\Psi(m_t)$. Thus, we proceed to show that $\Psi(m_t)$ is increasing and concave. First note that when $m_t = \hat{m}$:

$$\Psi(\hat{m}) = \hat{m}\left(1+\gamma\right), \tag{17}$$

where from equation (3) γ is given by:

$$\gamma = \left(\frac{1-\phi}{\phi}\right)\frac{\mu\left(\underline{h}\right)^{\delta-1}}{\left(\underline{h}\right)^{\delta}-h_t\left(1-\rho\right)},$$

and when m_t goes to infinity:

$$\lim_{m_t\to\infty}\Psi(m_t) = \infty. \tag{18}$$

If assumption 1 and 2 hold we have:

$$\Psi'(m_t) = \frac{1}{\phi}-\left(\frac{1-\phi}{\phi}\right)\left(\frac{\mu+h\left(m_t\right)-\underline{h}}{h'\left(m_t\right)}\right)\frac{h''\left(m_t\right)}{h'\left(m_t\right)} > 0, \tag{19}$$

and:

$$\Psi''(m_t) = \frac{h''\left(m_t\right)}{h'\left(m_t\right)}\left[\left(\frac{\mu+h\left(m_t\right)-\underline{h}}{h'\left(m_t\right)}\right)\left(\frac{2h''\left(m_t\right)}{h'\left(m_t\right)}-\frac{h'''\left(m_t\right)}{h''\left(m_t\right)}\right)-1\right] < 0, \tag{20}$$

since the term in the brackets is positive. In particular, it is given by:

$$\frac{(\mu + h_{t+1} - \underline{h})\left[\delta h_t (1-\rho) + \delta m_t^\delta\right] + m_t^\delta (\mu - \underline{h})}{m_t^\delta} > 0,$$

where we suppose that $\mu > \underline{h}$:

$$h_t > \underline{h}\left(1 + \frac{(\beta R)^{1/\phi}}{R}\right).$$

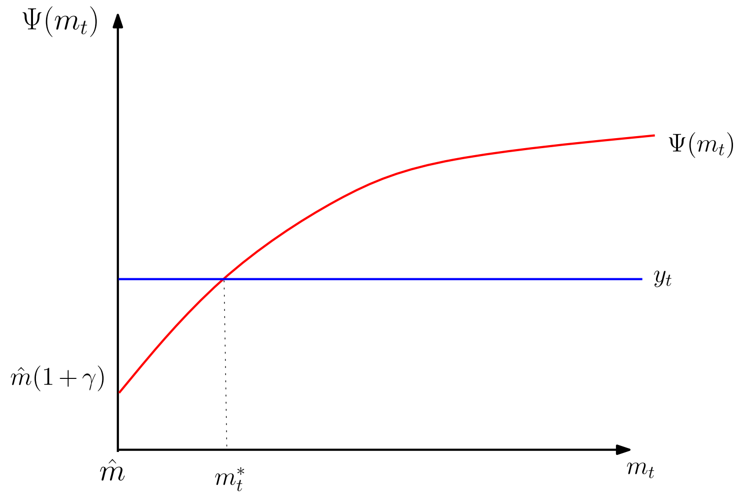

Figure 6: Existence of a solution.

Therefore, given equations (17) - (20), it follows that equation (12) has at least a real solution if (see figure 6):

$$y_t > \hat{m}(1+\gamma)$$

C Proof of proposition 2

Equation (12) implicitly defines optimal health investment as a function of income. Applying the implicit function theorem to equation (12) we get:

$$\frac{\partial m_t}{\partial y_t} > 0; \qquad \frac{\partial^2 m_t}{\partial y_t^2} > 0. \tag{21}$$

In particular, using equation (19) we have that:

$$\frac{\partial m_t}{\partial y_t} = \frac{1}{\Psi'(m_t)} > 0, \tag{22}$$

and from equation (20) we get:

$$\frac{\partial^2 m_t}{\partial y_t^2} = -\frac{\Psi''(m_t)}{\Psi'(m_t)} > 0. \tag{23}$$

Analysis of Health Share

From equation (12) the ratio between health investment and income is given by:

$$\frac{m_t}{y_t} = \frac{m_t}{\Psi(m_t)}, \tag{24}$$

from which:

$$\lim_{m\to\hat{m}} \frac{m_t}{y_t} = \frac{1}{1+\gamma} > 0, \tag{25}$$

and:

$$\lim_{m\to\infty} \frac{m_t}{y_t} = \phi$$

Given the optimal health investment we have that health share, increases in income if:

$$\frac{\partial(m_t/y_t)}{\partial y_t} = \frac{1}{y_t^2}\left[\frac{\partial m_t}{\partial y_t} y_t - m_t\right] > 0, \tag{26}$$

which implies that:

$$\varepsilon_m = \frac{(\partial m_t/\partial y_t)\, y_t}{m_t} > 1, \tag{27}$$

where ε_m is the elasticity of health spending with respect to income. Thus the health share behaves like a luxury good if it presents an elasticity with respect to income greater than one.

Using equations (12) and (19) we have:

$$\frac{\partial(m_t/y_t)}{\partial y_t} = \left(\frac{1-\phi}{\phi}\right)\left[\left(\frac{\mu + h(m_t) - \underline{h}}{h'(m_t)}\right)\left(1 + \frac{h''(m_t)}{h'(m_t)}\right) - m_t\right],$$

where the term in the brackets can be written as:

$$\frac{h(m_t)}{m_t^{1-\delta}\left[h_t(1-\rho) + m_t^{\delta}\right]^2}\left[m_t^{\delta}(\mu - \underline{h}) + \delta(1-\rho)\, h_t(\mu + h(m_t) - \underline{h})\right]$$

which positive if $\mu > \underline{h}$, that is:

$$h_t > \underline{h}\left(1 + \frac{(\beta R)^{1/\phi}}{R}\right).$$

D Proof of proposition 3

Given the relationship between saving and health in equation (11) we have that:

$$\lim_{m\to\hat{m}} s_t = 0 \tag{28}$$

and:

$$\lim_{m\to\infty} s_t = \infty \tag{29}$$

The relationship between saving and health spending is positive:

$$\frac{\partial s_t}{\partial m_t} = \left(\frac{1-\phi}{\phi}\right)\left[1-\left(\frac{h\left(m_t\right)-\underline{h}}{h'\left(m_t\right)}\right)\frac{h''\left(m_t\right)}{h'\left(m_t\right)}\right] > 0. \tag{30}$$

Analysis of Saving Share

The saving share behaves as follows:

$$\lim_{m\to\hat{m}} \frac{s_t}{y_t} = 0, \tag{31}$$

and:

$$\lim_{m\to\infty} \frac{s_t}{y_t} = 1-\phi.$$

Deriving the saving share with respect to income we get:

$$\frac{\partial(s_t/y_t)}{\partial y_t} = \frac{1}{y_t^2}\left(\frac{\partial s_t}{\partial m_t}\frac{\partial m_t}{\partial y_t}y_t - s_t\right) > 0. \tag{32}$$

We proceed to show that $\partial(s_t/y_t)/\partial y_t > 0$. From equations (16) and (22) we have that $\partial(s_t/y_t)/\partial y_t > 0$, if:

$$\frac{\partial s_t}{\partial m_t}\frac{1}{s_t} > \frac{\Psi'(m_t)}{\Psi(m_t)}, \tag{33}$$

where the function in the left behaves as follows:

$$\lim_{m\to\hat{m}} \frac{\partial s_t}{\partial m_t}\frac{1}{s_t} = \infty, \tag{34}$$

and:

$$\lim_{m\to\infty} \frac{\partial s_t}{\partial m_t}\frac{1}{s_t} = 0,$$

finally:

$$\frac{\partial\left(\frac{\partial s_t}{\partial m_t}\frac{1}{s_t}\right)}{\partial m_t} = \frac{1}{\left(s_t\right)^2}\left[\frac{\partial^2 s_t}{\partial m_t^2}s_t - \left(\frac{\partial s_t}{\partial m_t}\right)^2\right] < 0. \tag{35}$$

In particular, the explicit form of equation (35) is given by:

$$\frac{\partial\left(\frac{\partial s_t}{\partial m_t}\frac{1}{s_t}\right)}{\partial m_t} = -m_t^{2\delta}\left[h\left(m_t\right)\right]^2 - h_t\left(1-\rho\right)\left(1-\delta\right)\left[h\left(m_t\right)-\underline{h}\right]\left\{h_t\left(1-\rho\right)\left[h\left(m_t\right)-\underline{h}\right] + \right.$$
$$\left. + m_t^{\delta}\left[\left(2+\delta\right)h\left(m_t\right)-\left(1+\delta\right)\underline{h}\right]\right\}.$$

The function on the right side of equation (33) behaves as follows:

$$\lim_{m\to\hat{m}}\frac{\Psi'(m_t)}{\Psi(m_t)} = \frac{\frac{1}{\phi}+h_t\left(1-\delta\right)\left(1-\rho\right)\gamma\underline{h}^{-\delta}}{\hat{m}\left(1+\gamma\right)}, \tag{36}$$

and:

$$\lim_{m\to\infty}\frac{\Psi'(m_t)}{\Psi(m_t)} = 0,$$

finally:

$$\frac{\partial\left(\Psi'(m_t)/\Psi(m_t)\right)}{\partial m_t} = \frac{\Psi''(m_t)\Psi(m_t)-\left[\Psi'(m_t)\right]^2}{\Psi(m_t)^2} < 0,$$

from equations (19) and (20).

E Proof of proposition 4

The marginal utility of consumption and health is given by:

$$u_c = \left(1-\phi\right)\left(\frac{h\left(m_t\right)-\underline{h}}{c_{t+1}}\right)^{\phi},$$

$$u_h = \phi\left(\frac{c_{t+1}}{h\left(m_t\right)-\underline{h}}\right)^{1-\phi}.$$

Deriving both u_c and u_h with respect to income we get:

$$\frac{\partial u_c}{\partial y} = -m_t'(1-\phi)\phi\left(\frac{h\left(m_t\right)-\underline{h}}{c_{t+1}}\right)^{\phi}\left\{\frac{h_t\left(1-\rho\right)\left(1-\delta\right)}{\left[h_t\left(1-\rho\right)+\delta m_t^{\delta}\right]}\right\} < 0,$$

$$\frac{\partial u_h}{\partial y} = m_t'(1-\phi)\phi\left(\frac{h\left(m_t\right)-\underline{h}}{c_{t+1}}\right)^{\phi}\left\{\frac{h_t\left(1-\rho\right)\left(1-\delta\right)}{\left[h_t\left(1-\rho\right)+\delta m_t^{\delta}\right]}\right\} > 0,$$

The path of the ratio between saving and health spending depends on the following derivative:

$$\frac{\partial(s_t/m_t)}{\partial y_t} = \frac{1}{m_t^2}\frac{\partial m_t}{\partial y_t}\left[\frac{\partial s_t}{\partial m_t}m_t - s_t\right]$$

where from equation (22) $\partial m_t/\partial y_t > 0$. From equations (11) we get:

$$\frac{\partial s_t}{\partial m_t}m_t - s_t = \frac{\left(1-\phi\right)m_t}{\phi h\left(m_t\right)}\left\{\frac{\underline{h}}{\delta(1-\rho)h_t} - \frac{h\left(m_t\right)}{\delta h_t(1-\rho)+m^{\delta}}\right\}.$$

We study the sign of the two function in the brackets. In particular, we define the function on the left side as:

$$\Upsilon_1(m_t) = \frac{\underline{h}}{\delta(1-\rho)h_t} \tag{37}$$

and the function on the right side as:

$$\Upsilon_2(m_t) = \frac{h\,(m_t)}{\delta h_t(1-\rho) + m^\delta} \tag{38}$$

We proceed to analyze the function $\Upsilon_2(m_t)$:

$$\Upsilon_2(\hat{m}_t) = \frac{\underline{h}}{\underline{h}^\delta - h_t(1-\rho)(1-\delta)}, \tag{39}$$

and:

$$\lim_{m\to\infty} \Upsilon_2(m_t) = \infty. \tag{40}$$

$$\frac{\partial \Upsilon_2(m_t)}{\partial m_t} = \frac{m^{2\delta-1}(1-\delta)\left[h_t(1-\rho) + m^\delta\right]^{1/\delta-1}}{\left[\delta h_t(1-\rho) + m^\delta\right]^2} > 0 \tag{41}$$

From equation (39) we have that:

$$\Upsilon_1(m_t) > \Upsilon_2(\hat{m}_t), \tag{42}$$

since:

$$\underline{h}^\delta - h_t(1-\rho) > 0. \tag{43}$$

Therefore, there exists a value of $m_t = \overline{m}$ so that when $m_t < \overline{m}$ then $\Upsilon_1(m_t) > \Upsilon_2(m_t)$, and when $m_t > \overline{m}$ then $\Upsilon_1(m_t) < \Upsilon_2(m_t)$. In other words when $m_t < \overline{m}$ the ratio s_t/m_t increases and when $m_t > \overline{m}$ the ratio s_t/m_t decreases. Substituting $\overline{m}$ in equation (12), it follows that there exists a value $\overline{y}$ such that when $y_t < \overline{y}$ the ratio s_t/m_t increases and when $y_t > \overline{y}$ the ratio s_t/m_t decreases.

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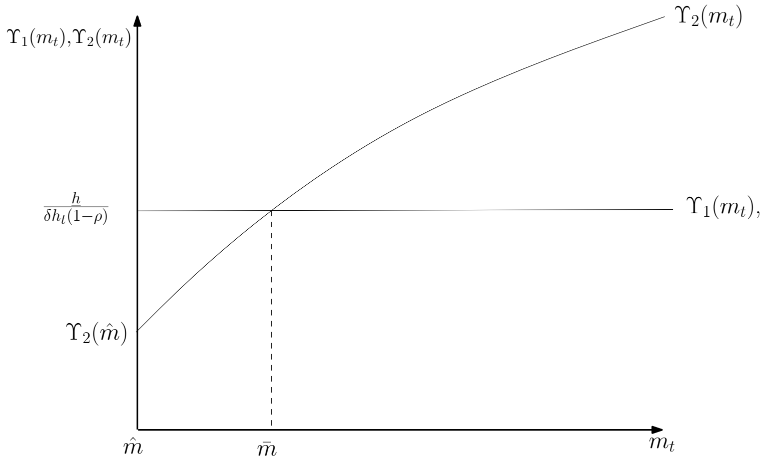

Figure 7: Analysis of the ratio between saving and health.

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