Centre for Household, Income, Labour and Demographic economics

Working Papers

PAYG Pensions and Human Capital Accumulation:

Some Unpleasant Arithmetic.

G.P. Cipriani and M. Makris

ChilD n. 19/2009

e-mail: de-child@unito.it
Web site: http://www.child-centre.it

PAYG Pensions and Human Capital Accumulation: Some Unpleasant Arithmetic*

Giam Pietro Cipriani
Department of Economics
University of Verona
Verona, Italy
giampietro.cipriani@univr.it

Miltiadis Makris
Department of Economics
University of Leicester
Leicester, UK
M.Makris@le.ac.uk

July 17, 2009

Abstract

A large literature has studied the effects of PAYG systems on fertility, human capital and growth. We argue that the social security system may also interact with longevity when the latter is endogenously determined. We show that in such an environment, in a dynamically efficient economy PAYG pensions must be sufficiently low in order to ensure positive economic growth. Moreover, a transition to a funded social security system will promote growth, and can thereby take place by fully compensating the losers.

JEL Classification: H55, J10, O10

Keywords: Pensions, Human Capital, Growth, Endogenous Longevity.

*Correspondence: Dr. Miltiadis Makris, Department of Economics, University of Leicester, University Road, Leicester LE1 7RH, E-mail: M.Makris@le.ac.uk., TEL: 0044 (0) 116 2525645. We would like to thank Pierre Pestieau and Dimitrios Varvarigos for useful discussions and comments. The usual disclaimer applies.

1 Introduction

The well-known demographic changes projected throughout this century have raised a debate about the sustainability of Pay-As-You-Go pension systems, which still operate in most developed countries. These systems were designed in a world in which few people survived to many retirement years and fertility rates were much higher than today. With falling fertility, increasing longevity and no substantial change in the retirement age, the PAYG system comes under strain. The central issue in the political debate is whether and how to shift from a PAYG scheme towards a system with a higher weight on the funded component of social security to ensure that the system will be sustainable in the long run despite the changing population structure.

It is not surprising then, that a vast literature has studied the interaction between public pensions and the fertility rate. In fact, PAYG pensions have been shown to distort family choices on fertility as well as on human capital accumulation (and, of course, savings). The basic source of these distortions is that each individual's fertility decision affects the overall rate of return of PAYG benefits, but this is not taken into account by optimizing agents. Also, since an individual's pension entitlement is independent of his children's "quality", she has no incentive to consider the marginal social benefit conferred through their contribution to their children's human capital. These effects have been the subject of a literature that has considered pensions in overlapping generations models with endogenous fertility. Examples include Cigno (1992), Ehrlich and Lui (1998), Nishimura and Zhang (1992), Rosati (1996), Wigger (1999) and, more recently, Cremer et al. (2006) and Alders and Broer (2005). Another demographic factor related to the pension system is the process of family formation itself, as recently shown by Ehrlich and Kim (2005). In their work, it is stressed that social security systems can create adverse incentives on family formation and, consequently, on fertility, human capital accumulation and savings.

On the other hand, the other important determinant of the demographic change, longevity, is usually not included in the analysis and when it is included it is usually considered as exogenous (see, for example, Echevarria and Iza, 2006 and Andersen, 2008). However, as a well-established literature has made clear, longevity can be affected by individual choices and government policies. Hence, in this paper we focus on longevity and human capital and treat fertility as exogenous. This does not imply, of course, that fertility is less important than longevity. Only, that this channel has already been explored in theoretical models of social security and growth, and that we would like to make our point in a transparent way.

We build up a three-period overlapping generations model where agents decide on their own educational effort, which determines, together with inherited human capital, their available stock of knowledge. Old agents consume their savings and the proceeds of a PAYG pension. In addition, like in Blackburn and Cipriani (2002), Lagerlöf (2003) and Cervellati and Sunde (2005), we assume that longevity of a generation depends on the average human capital level of that generation to reflect the fact that better educated individuals and their social networks, in the presence of social spillovers, are more likely to adopt healthy life-styles.[1] We focus on the consequences for human capital accumulation and thereby growth of PAYG pensions.[2] We then show that under exogenous longevity there exists a PAYG system so that investment in education and hence growth are always positive. However, when longevity depends on human capital, the equilibrium, if it exists for this given PAYG system, may display zero accumulation of human capital and zero (endogenous) growth. To understand this, note that any pension scheme affects agents' wealth and thereby the private accumulation of human capital, which in turn affects longevity. Importantly, however, the latter effect may be in a way which is inconsistent with the projected path of longevity used to design pensions in the first place. In fact, if pensions are sufficiently high and decreasing with life expectancy, then zero accumulation of human capital will be the unique equilibrium in a dynamically efficient economy. Finally, in our model, like in other endogenous growth models and contrary to the traditional result, it is possible to set up a Pareto improving transition to a fully funded system in a dynamically efficient economy. In fact, the government can borrow to compensate the initial adult population (who pays contributions but will not receive a pension) given that this pension policy change will spur growth by affecting human capital investment.

The effects of an unfunded pension scheme on human capital have also been studied by Ehrlich and Lui (1998) and Sinn (2004). These papers show that a PAYG system has an adverse effect on parental investment on their children's human capital. However, the effects of social security on own human capital investment, which are our focus here, have received very little attention. Recent exceptions include Poutvaara (2007) and Echevarria and Iza (2006). The former analyses an open economy with migration where countries differ in their pension system. The latter consider a model where PAYG pensions depend on the contributions made by individuals during their working lives. In these papers, human capital investment is affected by the pensions system. However these papers do not include longevity in the analysis. Our paper fills a gap in the literature by considering the interactions between an unfunded pension scheme, human capital accumulation and *endogenous* longevity. Here, the pension system affects the incentives to accumulate own human capital which, in turn, affect longevity and therefore the operation of the pension system itself. In this setting, we show that if pensions are sufficiently high and decreasing with life expectancy, then the economy may fall in a trap with no human-capital-driven growth.

Similarly to this paper, Cipriani and Makris (2007) also consider endogenous longevity. However

[1] As human capital is the key to growth in this paper, the dependence of longevity on human capital builds a link between growth and longevity. For empirical evidence on the relation between education and health see for example Furnée et al. (2008). For evidence on the relationship between longevity and growth see Barro and Sala-i-Martin (2004).

[2] Thus, we do not investigate social welfare maximising pension policies. For a work along these lines, but with exogenous longevity, see Docquier et al. (2007). The implicit assumption behind our focus here is that social welfare maximising policies might not emerge as a result of political failures.

in that paper a wider range of intergenerational redistributive policies are considered. Moreover, in their work growth is always positive and the focus is on local indeterminacy of dynamic equilibria. Here, instead, there can be no local indeterminacy, while growth depends on the particulars of the PAYG system. There are few other papers concerned with endogenous longevity and pension programs. The seminal paper in this literature, Philipson and Becker (1998), shows that when income earned is contingent on one's length of life there is a moral hazard effect that induces excessive longevity. For a discussion along these lines see also Davies and Kuhn (1992). Finally, Pestieau et al. (2006) consider explicitly the issue of subsidising or taxing private health expenditure that affects longevity in an economy with a PAYG system. In all these papers, differently from our approach, there is no human capital accumulation or growth.

This paper is organised as follows. Section 2 introduces the model. Section 3 develops the equilibrium. Section 4 presents some benchmark cases. Section 5 analyses the "unpleasant arithmetic" case of a zero growth (unique) equilibrium. Section 6 discusses our results and their implications for a switch to a funded pension system and concludes.

2 The Model

The model consists of an economy with three overlapping generations in each period t, with $t = 1, ..., \infty$. As a matter of convention we will identify generations by the time agents were born. Thus, a superscript t will denote variables that are associated with agents born in period t. We will also be using time-subscripts for aggregate variables. That is, a subscript t will denote an aggregate variable of period t.

2.1 Firms

Assume perfect competition in the product market. Output in every period t is produced by means of a constant returns to scale production function that uses physical capital, K_t, and effective labour, L_t, denoted by $F(K_t, L_t)$, with $F_1 > 0, F_{11} < 0, F_2 > 0, F_{22} < 0$. Effective labour is the total labour supply in terms of time-endowment adjusted by the productivity of each unit of labour-time supplied. This productivity depends on the human capital possessed by workers. Let $k \equiv K/L$ be the physical capital stock per-unit of effective labour, and $f(k)$ the intensive form representation of the production function, i.e. $f(k) \equiv F(\frac{K}{L}, 1)$. Assume for simplicity that capital depreciates fully after production takes place.

Let R_t be the gross real interest rate and w_t the real wage per unit of effective labour. Profit-maximisation implies that

$$f'(k_t) = R_t, \text{and} \quad (1)$$

$$f(k_t) - f'(k_t)k_t = w_t. \quad (2)$$

Notice that in this model output in period t is given by $L_t f(k_t)$. Therefore the growth rate of output will be driven by the growth of effective labour, L_t, and the growth of physical capital per effective labour. To deliver our message in a simple and transparent way, we assume hereafter that the interest rate is exogenously given (this would be the case, for instance, in a small open economy) and time-invariant, i.e. $R_t = R$ for any t. This in turn implies that capital per effective labour and the effective wage are determined at the world level as $f'^{-1}(R) \equiv k$ and $f(k) - kf'(k) \equiv w$, respectively. For the same reason we assume that the supply of labour-hours is also exogenous. These two assumptions imply that the growth of output is driven purely by the growth of effective labour (and hence, as we shall shortly see, by population growth rate and the evolution of human capital over time), but, crucially, not by domestic savings and/or the world market for capital.

2.2 Households

Agents belong to overlapping generations with finite but uncertain lifetimes. Denote by N_t the number of born agents in period t. These will be the adults in period $t+1$ and the old agents in period $t+2$. Denote by $n^{t-1} \geq 1$ the number of children of each and every adult in period t. We thus have $N_t = N_{t-1} n^{t-1}$. Given that fertility is exogenous in this model, for simplicity of exposition assume that $n^t \equiv n$ at any time t.

Each agent matures safely from childhood to adulthood and has a probability of surviving to old age. In the first period of life an agent is raised by her parent and decides how much *time/effort* to invest in her education or to devote to leisure. When adult he becomes a producer and consumer of output. If she survives to old age, an agent just consumes her income from savings and pensions. The endowment of time, available for labour, in each period is normalised to one. Adult agents spend their whole labour-time-endowment on working, and agents who survive in old age spend their whole time-endowment in retirement/leisure. All agents have identical preferences and technologies, and are aware of their life expectancies.

The expected lifetime utility of an agent of generation t is thus given by

$$W^t = U(1 - e^t) + \delta\phi(c_a^t) + \pi^t \delta^2 \phi(c_o^t) \tag{3}$$

where e^t denotes time/effort spent in education when young. Moreover, π^t is the probability of surviving to the third period, or longevity. c_o^t denotes the consumption when old (i.e. in period $t+2$) and c_a^t denotes consumption when adult (i.e. in period $t+1$). The functions U and ϕ are twice continuously differentiable, strictly increasing and concave, and satisfy the Inada conditions. Finally, $\delta \in (0, 1)$ denotes the discount factor.

Each agent enters her second period of life with an amount of human capital, h^t, which determines, in a one-to-one way, the agent's labour productivity. It also affects the general 'knowledge' of the agent and (through social interactions and spillovers) of her generation. Recall that in our model only adult agents work. Accordingly, in equilibrium, total effective labour in period $t+1$ is given by

$$L_{t+1} = N_t h^t.$$

The human capital of an agent born in period t is partly inherited from her parent and partly the result of her own educational effort. In particular,[3]

$$h^t = h^{t-1}[1 + \mu(e^t)] \tag{4}$$

where $\mu(e)$ is twice continuously differentiable with $\mu(0) = 0$, $\mu' > 0, \mu'' \leq 0$, and $\mu'(0) \equiv \gamma$ being finite. That is, the human capital growth rate is a strictly increasing and weakly concave function of investment in education with finite returns. The case of $\lim_{e \to 0} \mu'(e) = \infty$ is discussed as a benchmark case shortly. Denote also with $h^0 > 0$ the inherited human capital of the typical agent born at time $t = 1$. Given our discussion, we have that in our set up the economy is growing over and above its exogenous growth rate if and only if education effort is positive. In fact, the output growth rate in period t is $n(1 + \mu(e^t)) - 1$.

The consumption level of a generation-t adult agent with human capital h^t is

$$0 \leq c_a^t = (1 - \tau^t - s^t) w h^t \tag{5}$$

where $1 > \tau^t \geq 0$ is an income tax-rate (administered in period $t+1$), the proceeds of which are used to finance transfers to old agents, i.e. PAYG pensions. Also, s^t denotes, as a proportion of the agent's effective wage, the sum of private net savings, any purchases of arbitrage-free public debt and any compulsory contributions towards a balanced government-run pension fund.

Agents transact in the capital market through intermediaries. These intermediaries are infinite-horizon entities that face the risk-free interest rate R. This is also the interest rate at which intermediaries borrow from or lend to the government - assumed also to be an infinitely-lived entity. When, however, intermediaries lend to or borrow from individuals, the corresponding rate must incorporate the risk involved due to the agents' uncertain lifetimes. Assuming, then, that intermediaries operate under conditions of perfect competition, and that entry is costless, we have that this rate of return in period $t+2$ is equal to R/π^t. It follows that consumption in old age is

$$0 \leq c_o^t = \left[p^t + s^t (R/\pi^t)\right] w h^t \tag{6}$$

where $p^t \geq 0$ is the PAYG pension (received in period $t+2$), as a proportion of labour income in

[3] Note that our results are qualitatively robust to allowing for a given path of public spending on education that complements the education that takes place in a household which is the focus of our study.

adulthood.

It will prove to be useful to derive the intertemporal budget constraint of the typical member of generation t. We have that

$$\begin{aligned} c_a^t + \frac{\pi^t c_o^t}{R} &= wh^t(1 - \tau^t + \frac{p^t \pi^t}{R}) \qquad (7) \\ &\equiv wh^t(1 + G^t) \\ &\equiv m^t \qquad (8) \end{aligned}$$

Note that $G^t > -1$ is assumed to ensure positive consumption levels. G^t represents the net benefit, in terms of wealth m^t, for generation t of the existence of PAYG pensions. This *net pension benefit* takes into account the fact that agents are both contributors to and claimants of such pensions.

We turn to the specification of one important feature of our model which is the endogenous determination of the survival probability, π. Like in Blackburn and Cipriani (2002) and in Lagerlöf (2003) we assume that longevity depends on human capital. In particular, we assume that the longevity of generation t depends on the average human capital level of that generation.[4] This assumption is in line with a large body of empirical evidence recently surveyed by Cutler et al. (2006). This literature finds that the amount of human capital available in society is an important determinant of longevity. This result is also confirmed by empirical research concerning developed economies. For example a recent paper by Wheeler (2008) finds, in a sample of 226 U.S. metropolitan areas, a significant effect of average human capital on mortality, after conditioning on various individual-specific characteristics including income and own education. Hence, as other papers in the literature, like Tamura (2006) and Cipriani and Makris (2007), we assume that average human capital affects longevity. That is,

$$\pi^t = \pi\left(\overline{h}^t\right) \qquad (9)$$

where $\overline{h}^t$ is the average level of human capital of generation t, with $\pi'(.) > 0$, $\pi(0) = \underline{\pi}$ and $\lim_{\overline{h} -> \infty} \pi\left(\overline{h}\right) = \overline{\pi} \leq 1$. Let also $\pi^0 \equiv \pi(h^0)$, and recall that h^0, and hence π^0, is pre-determined.

The typical agent born in period t is faced with the problem of maximising (3) with respect to e^t, c_a^t and c_o^t subject to (4), (7) and $e^t \in [0, 1]$ taking as given the net pension benefit, G^t, inherited human capital, h^{t-1}, longevity, π^t, and prices w and R.

Note that in equilibrium $\overline{h}^t = h^t$ and that the equilibrium paths for e^t, c_a^t, c_o^t, s^t, h^t, π^t, follow from the solution to the above problem, (5), (9) and $\overline{h}^t = h^t$.

[4] We could also assume that longevity depends additionally on personal human capital. However, while this assumption would complicate the analysis, it would not change our main result as long as average human capital affects longevity.

2.3 The PAYG system

Turning to pension policy, notice that in our set-up individuals differ in equilibrium *only* in terms of their age and the longevity of their generation. Their age determines whether they are recipients or contributors of PAYG pensions. The longevity (and hence human capital) of a generation is a measure of its contribution to the economy's growth - given inherited human capital. It is also a measure of the total burden imposed on the next generation due to the presence of PAYG pensions - given population growth, the tax and the effective wage (and hence the interest rate). For these reasons we assume that the period$-t+2$ pension rate, p^t, faced by a member of generation t depends only on longevity π^t and the fertility rate and effective wage. To capture this, we assume that p^t is given by some policy function $p(\pi^t) \equiv p(\pi^t, n, w)$. As we will see, the monotonicity properties of $p(\pi^t)$ and the initial pension rate $p(\pi^0)$ will be important for the implications of pensions for human capital accumulation

3 Equilibrium Characterisation

3.1 Individual Choices

After eliminating c_a^t from the utility function, by using the intertemporal budget constraint, the first order conditions (FOCs) with respect to c_o^t and e^t are:

$$R\delta\phi'(c_o^t) = \phi'(c_a^t), \tag{10}$$

and

$$\delta w h^{t-1}(1+G^t)\phi'(c_a^t)\mu'(e^t) + \zeta^t = U'(1-e^t), \tag{11}$$
$$\zeta^t e^t = 0,\ \zeta^t \geq 0, e^t \geq 0,$$

respectively, where ζ^t is the Kuhn-Tucker multiplier of the non-negativity constraint on education effort. Notice that by the Inada conditions $e^t < 1$. The first condition is the solution of the standard trade-off between current and future consumption. The last FOC is the resolution of the trade-off between leisure when young and labour income when adult - and hence consumption in adulthood and seniority.

Using the intertemporal budget constraint and the first FOC above we can find the optimal consumption in adulthood. It follows, after suppressing, for expositional clarity, its dependence on δ and R, that optimal consumption in adulthood for members of generation t is given by a function $c_a(\pi^t, m^t)$ which is strictly increasing in wealth and strictly decreasing in longevity ceteris paribus.[5]

---

[5] These follow directly after solving implicitly for c_o^t from (9) and using this solution into the intertemporal budget constraint.

Turning to the effort-choice, let us define $C(e^t) \equiv \frac{U'(1-e^t)}{\mu'(e^t)}$, the marginal utility-cost of an extra unit of growth rate of human capital. Also, $B(e^t, wh^{t-1}, \pi^t, G^t) \equiv \delta wh^{t-1}(1+G^t)\phi'(c_a(\pi^t, wh^{t-1}(1+\mu(e^t))(1+G^t)))$ is the marginal welfare benefit from an extra unit of growth rate of human capital. Note that $U'' < 0$, $U' > 0$, $\mu' > 0$ and $\mu'' \leq 0$ imply that $C(e)$ is strictly increasing in e, with $\lim_{e\to 1} C(e) = \infty$ due to the Inada conditions. Note also that $\phi'' < 0$, $\partial c_a/\partial m > 0$ and $\mu' > 0$ imply that $B(e^t, wh^{t-1}, \pi^t, G^t)$ is strictly decreasing in e. We then have that at optimum, if $B(0, wh^{t-1}, \pi^t, G^t) \leq C(0)$ then optimal effort is zero, while if $B(0, wh^{t-1}, \pi^t, G^t) > C(0)$ then optimal effort is strictly positive and given implicitly by $B(e^t, wh^{t-1}, \pi^t, G^t) = C(e^t)$. Let

$$e^t = e(wh^{t-1}, \pi^t, G^t) \tag{12}$$

be the optimal education effort as a function of inherited human capital, h^{t-1}, (rationally) anticipated longevity, π^t, and net pension benefit, G^t.

Recalling the properties of $\phi(.)$ and $c_a(.)$, note that B_3 is strictly positive. This follows from the fact that higher longevity reduces consumption in adulthood (for given wealth) and hence it increases the marginal welfare benefit of higher labour income and thereby consumption. Therefore, in this case, when $e_t > 0$, we have that $e_2 > 0$: higher rationally anticipated longevity increases investment in education.

B_2 and B_4, however, cannot be signed unless we put further restrictions on preferences of consumption. The reason is that either of h^{t-1} and G^t by increasing wealth have a direct positive effect on B, by increasing consumption in adulthood for given marginal utility of consumption, as well as an indirect negative effect on B by decreasing the marginal utility of consumption when adult. So the effects of inherited human capital and net pension benefit on education effort are ambiguous in general. In what follows we focus on the case of $\sigma(c_a)\varepsilon^a_m < 1$, where $\sigma(c) \equiv -\frac{c\phi''(c)}{\phi'(c)}$ is the elasticity of substitution and $\varepsilon^a_m \equiv \frac{m\partial c_a}{c_a \partial m}$ is the wealth-elasticity of consumption in adulthood. This ensures that $B_2 > 0$ and $B_4 > 0$, and thereby, when $e_t > 0$, that $e_1 > 0$ and $e_3 > 0$: higher inherited human capital and/or net pension benefits lead to higher investment in human capital.[6] With this assumption we do not disadvantage growth a priori by the presence of PAYG pensions. Moreover, human capital accumulation has a positive intergenerational spillover effect, i.e. there is a growth propagation effect. Notice that this assumption would be satisfied with a CES utility from consumption, where we have $\sigma(c) = \sigma < 1$ and $\varepsilon^a_m = 1$.[7]

Assume hereafter that $B(0, wh^{t-1}, \pi^t, 0) \equiv \delta wh^0\phi'(c_a(\pi^0, wh^0)) > U'(1)/\gamma \equiv C(0)$ and hence

[6] After some straighforward manipulations we have that $B_2(e^t, wh^{t-1}, \pi^t, G^t) = \delta w(1+G^t)\phi'(c^t_a)[1-\sigma(c_a)\varepsilon^a_m]$ and $B_4(e^t, wh^{t-1}, \pi^t, G^t) = \delta wh^{t-1}\phi'(c^t_a)[1-\sigma(c_a)\varepsilon^a_m]$.

[7] It would not hold however if utility was logarithmic, where $\sigma(c) = \varepsilon^a_m = 1$. In this case, we would have $B_2 = B_4 = 0$ and hence $e_1 = e_3 = 0$. The logarithmic case is discussed as a benchmark case shortly.

$e(wh^0, \pi^0, 0) > 0$. Thus, after recalling that consumption in adulthood is strictly decreasing with longevity for given wealth, with any path of non-decreasing longevity over time we have that, in the absence of PAYG pensions, effort is strictly positive in any period.

Notice that $\lim_{G\to\infty} e(wh^{t-1}, \pi^t, G) = 1$. Note also that if $G^t = -1$ then consumptions are zero and education has no effect on wealth and thereby welfare of members of generation t. So, optimal education effort in period t is zero. Thus, if the pension scheme imposes a sufficiently high loss on households, then optimal effort is zero. In other words, for sufficiently low net pension benefit, optimal effort is zero. Denote this threshold level by $G(wh^{t-1}, \pi^t)$, and note that it is given implicitly by the solution with respect to G^t of $\delta wh^{t-1}(1+G^t)\phi'(c_a(\pi^t, wh^{t-1}(1+G^t))) = U'(1)/\gamma$. Thus, $G(wh^{t-1}, \pi^t)$ is strictly decreasing in both wh^{t-1} and π^t. Note that, by our assumptions so far, $-1 < G(wh^{t-1}, \pi^t) < 0$ for any $t \geq 1$.

3.2 Equilibrium Pensions

By the definition of PAYG pensions we have

$$w_{t+1}\bar{h}^t N_t \tau^t = w_t \bar{h}^{t-1} N_{t-1} \pi^{t-1} p^{t-1}. \tag{13}$$

In equilibrium, after recalling (4), $w_{t+1} = w_t$, $N_t = nN_{t-1}$ and $\bar{h}^t = h^t$ for any t, we thus have

$$\tau^t = \frac{p^{t-1}\pi^{t-1}}{n(1+\mu(e^t))}. \tag{14}$$

So, the net pension benefit is given by

$$G^t = \frac{p^t \pi^t}{R} - \frac{p^{t-1}\pi^{t-1}}{n(1+\mu(e^t))}. \tag{15}$$

Note that the net pension benefit is strictly increasing with effort. Let, for latter use,

$$\Delta^t(\pi^t) \equiv \frac{p(\pi^t)\pi^t}{R} - \frac{p(\pi^{t-1})\pi^{t-1}}{n} \tag{16}$$

be the equilibrium net pension benefit of the typical member of generation t when rationally anticipated longevity is π^t and education effort is zero.

3.3 Equilibrium Longevity

Recall that in equilibrium $\overline{h}^t = h^t = h^{t-1}(1+\mu(e^t))$. Let $\kappa(.)$ be the inverse of $\pi(.)$. That is, $h^t = \kappa(\pi^t) \equiv \pi^{-1}(\pi^t)$. We then have that in equilibrium

$$\kappa(\pi^t) = (1+\mu(e^t))\kappa(\pi^{t-1}). \tag{17}$$

This determines a law-of-motion for endogenous longevity $\pi^t = \Pi(\pi^{t-1}, e^t)$ with $\Pi(\pi, 0) = \pi$, $\Pi(\bar{\pi}, e) = \bar{\pi}$, and, finally, $\Pi(\pi, e) > \pi$ if $\pi < \bar{\pi}$ and $e > 0$.

Equilibrium is determined by the solution to the system of (12) - after using $h^{t-1} = \kappa(\pi^{t-1})$, (15) and (17), with human capital and consumption levels following residually from $h^t = \kappa(\pi^t)$, the intertemporal budget constraint, (7), and the FOC with respect to c_o^t, (10). Before we move to a more detailed investigation of equilibrium, we discuss next some benchmark cases that will help the better understanding of our forthcoming results.

4 Benchmark Cases

First, note that if $\lim_{e\to 0} \mu'(e) = \infty$ then $\lim_{e\to 0} C(e) = 0$. Therefore, for any net pension benefit $G^t > -1$, we have that investment in education is positive in every period. Thus, the economy grows permanently for any PAYG pension scheme, interest rate and adult-population growth rate.

Second, note that if $\mu'(0) = \gamma$ is finite but $\phi(c) = \log c$ then $\sigma(c_a)\varepsilon_m^a = 1$ and education effort is independent of inherited human capital and, crucially, the net pension benefit. It follows directly that our assumption that $\delta w h^0 \phi'(c_a(\pi^0, wh^0)) > U'(1)/\gamma$ implies, conditional on existence of equilibrium, that human capital accumulation is positive in every period.

Finally, return to the case of $\mu'(0) = \gamma$ being finite and $\sigma(c_a)\varepsilon_m^a < 1$, and consider an environment with exogenously increasing longevity. Suppose also that PAYG pensions are designed so that given this longevity path, investment in education is positive for any period. Formally, fix π^t and π^{t-1} such that $\pi^t > \pi^{t-1}$. Suppose also that, for these longevities, the PAYG system satisfies

$$\Delta^t(\pi^t) > G(wh^{t-1}, \pi^t) \tag{18}$$

.This implies that the system of (12) and (15) has only solutions with positive effort, if longevity of generation t is expected to be π^t and $\pi^{t-1} < \pi^t$. Denote such a solution by $\hat{e}^t$. Figure 1 illustrates.

Add Figure 1 here

The above scenario is intended to capture a situation where the government uses some projection of the path of longevity to design PAYG pensions in a way that growth is not hindered. However, as our forthcoming analysis will emphasise, this will not be enough if longevity is, instead, endogenous. The reason is simple: any pension scheme affects the private accumulation of human capital, which in turn affects longevity, possibly in a way inconsistent with the projected path of longevity used to design pensions. This echoes similar results in the literature on endogenous fertility where the PAYG scheme adversely affects fertility exacerbating the risk of financial collapse of the system itself.

We now turn to our focus: the case of endogenous longevity with $\mu'(0) = \gamma$ being finite and $\sigma(c_a)\varepsilon_m^a < 1$.

5 The Unpleasant Arithmetic

With endogenous longevity, the equilibrium is, in effect, characterised by the solution to the system of (12) - with $h^{t-1} = \kappa(\pi^{t-1})$, (15) and (17). It will be helpful to manipulate this system as follows. First, rewrite (17) as $e^t = \mu^{-1}(\frac{\kappa(\pi^t)}{\kappa(\pi^{t-1})} - 1)$ where μ^{-1} is the inverse of μ. This is the effort level consistent with past and rationally expected longevities π^{t-1} and π^t, respectively. Second, using (17), rewrite (15) as $G^t = \Gamma^t(\pi^t)$, where we have defined $\Gamma^t(\pi^t) \equiv \frac{p(\pi^t)\pi^t}{R} - \frac{\kappa(\pi^{t-1})p(\pi^{t-1})\pi^{t-1}}{n\kappa(\pi^t)}$. The latter is the generation-t net benefit from the pension system that is consistent with past and rationally expected longevities π^{t-1} and π^t, respectively. Using these expressions for equilibrium educational investment and generation-t net benefit, we have that the private marginal benefit and the marginal cost from an extra unit of growth rate of human capital are given by

$$\hat{B}^t(\pi^t) \equiv B(\mu^{-1}(\frac{\kappa(\pi^t)}{\kappa(\pi^{t-1})} - 1), w\kappa(\pi^{t-1}), \pi^t, \Gamma^t(\pi^t)) \tag{19}$$

and

$$\hat{C}^t(\pi^t) \equiv C(\mu^{-1}(\frac{\kappa(\pi^t)}{\kappa(\pi^{t-1})} - 1)), \tag{20}$$

respectively. Recalling, then, our discussion of the private incentives to exert educational effort (see the discussion of condition (11)), we have that an equilibrium with $\pi^t > \pi^{t-1}$, and hence with positive generation-t effort, is characterised by $\hat{B}^t(\pi^t) = \hat{C}^t(\pi^t)$. An equilibrium with zero education effort in period t, and hence $\pi^{t-1} = \pi^t$, exists, on the other hand, if and only if $\hat{B}^t(\pi^{t-1}) \leq \hat{C}^t(\pi^{t-1})$.

Note next that with endogenous longevity there is no guarantee that an equilibrium exists. That is, given our discussion above, it could be the case that $\hat{B}^t(\pi^{t-1}) > \hat{C}^t(\pi^{t-1})$ and that there is no solution to $\hat{B}^t(\pi^t) = \hat{C}^t(\pi^t)$ such that $0 < \pi^t \leq 1$. This can be the case even if there is a solution for some π^t and $\pi^{t-1} \leq \pi^t$ to the system, discussed in Section 4, of (12) and (15), after setting $\pi^{t-1} \equiv \pi(h^{t-1})$ and hence $h^{t-1} = \kappa(\pi^{t-1})$. The reason is clear: it may be the case that $\pi^t \neq \pi(\kappa(\pi^{t-1})(1 + \mu(\hat{e}^t)))$, where $\hat{e}^t$ is, recall, a positive effort level that is part of a solution of (12) and (15), given some π^t and π^{t-1}. To ensure existence of equilibrium in period t, assume hereafter that $\hat{B}^t(1) \leq \hat{C}^t(1)$.

More importantly, even if PAYG pensions are such that $\pi^t = \pi(\kappa(\pi^{t-1})(1 + \mu(\hat{e}^t)))$, and hence $\hat{e}^t$ is an equilibrium even under endogenous longevity, we have that zero effort may also be an equilibrium. Given our discussion above, this will be the case if $\hat{B}^t(\pi^{t-1}) < \hat{C}^t(\pi^{t-1})$. In more detail, note that in such an equilibrium anticipated longevity of generation t will be the longevity of the previous generation, $\pi^t = \pi^{t-1}$ (recall (17)). Thus, the net pension benefit of generation t will be $G^t = \Gamma^t(\pi^{t-1}) = \Delta^t(\pi^{t-1})$, where the latter equality follows directly from the definition of $\Gamma^t(.)$ above and of $\Delta^t(.)$ in Section 3.2 (see (16)). Moreover, we have that $\mu^{-1}(0) = 0$ and hence $\hat{B}^t(\pi^{t-1}) = B(0, w\kappa(\pi^{t-1}), \pi^{t-1}, \Delta^t(\pi^{t-1}))$ and $\hat{C}^t(\pi^{t-1}) = C(0)$. Clearly, then, given the definition of $G(.)$, zero effort in period t will be an equilibrium if, given $\kappa(\pi^{t-1}) = h^{t-1}$,

$$\Delta^t(\pi^{t-1}) \leq G(w\kappa(\pi^{t-1}), \pi^{t-1}). \tag{21}$$

To understand how the above could arise, recall first that $G(wh^0, \pi^0) < 0$. Note also that $\Delta^t(\pi^{t-1})$ $= \pi^{t-1}p(\pi^{t-1})[\frac{n-R}{nR}]$. So, if $R > n$ then $\Delta^1(\pi^0) < 0$. It follows then directly that if the economy is dynamically efficient, i.e. $R > n$, and the "initial" pension rate, $p(\pi^0)$, is sufficiently high so that $G(wh^0, \pi^0) > \pi^0 p(\pi^0)[\frac{n-R}{nR}]$, then we have that (21) holds for sufficiently low past longevity, π^{t-1}, regardless of the monotonicity of $\pi p(\pi)$ and $G(w\kappa(\pi), \pi)$ with respect to π. Formally, (21) is true for any $\pi^0 \leq \pi^{t-1} \leq \tilde{\pi}$, where $\tilde{\pi} > \pi^0$ is the lowest longevity π for which $\pi p(\pi)[\frac{n-R}{nR}] \geq G(w\kappa(\pi), \pi)$, with $\tilde{\pi} \equiv \bar{\pi}$ if the latter inequality does not hold for every $\pi^0 \leq \pi \leq \bar{\pi}$.

Note now that (21) could a priori hold even if (18) holds, i.e. even if $\Delta^t(\pi^t) > G(wh^{t-1}, \pi^t)$. To see this, recall that $G(wh^{t-1}, \pi^t)$ is strictly decreasing on π^t. Note then that, for any $\pi^t > \pi^{t-1}$, we have $G(wh^{t-1}, \pi^{t-1}) > G(wh^{t-1}, \pi^t)$, and if $\pi^t p(\pi^t) > \pi^{t-1}p(\pi^{t-1})$ then $\Delta^t(\pi^t) > \Delta^t(\pi^{t-1})$. So, if the pension system is such that, given π^t and $\pi^{t-1} < \pi^t$, we have that $\pi^t p(\pi^t) - \pi^{t-1}p(\pi^{t-1})$ is sufficiently large, then (18) and (21) can be compatible. Figure 2 illustrates.

Add Figure 2 here

More interestingly, we can also have an environment where zero human capital accumulation is the only equilibrium under endogenous longevity. Recall from above that zero effort is an equilibrium if

$$\hat{B}^t(\pi^{t-1}) = B(0, w\kappa(\pi^{t-1}), \pi^{t-1}, \Delta^t(\pi^{t-1})) \leq C(0) = \hat{C}^t(\pi^{t-1}). \tag{22}$$

Note also that $\hat{C}^t(\pi)$ is strictly increasing with π, due to the properties of $\mu(.)$, $\pi(.)$ and $C(.)$. Therefore, it follows directly that no human capital accumulation will be the unique equilibrium if $\hat{B}^t(\pi^t)$ is non-increasing. To see how this can be the case, recall first that $B_4 > 0$ and note thus, after some straightforward differentiation and use of the definitions of B and Γ^t, that $\hat{B}^{t\prime}(\pi^t)$ is strictly increasing in $\frac{d[p(\pi^t)\pi^t]}{d\pi^t}$. Let us refer to a PAYG pension system as elastic if $\pi p(\pi)$ is strictly decreasing in π. We then have directly, and as a means of summarising our discussion in this section, that:

Proposition: *If the economy is dynamically efficient, the PAYG initial pension rate is sufficiently high and longevity of the previous generation is sufficiently low, so that $\Delta^t(\pi^{t-1}) \leq G(w\kappa(\pi^{t-1}), \pi^{t-1})$, then zero human capital accumulation in period t is an equilibrium. If, in addition, the PAYG pension system is sufficiently elastic so that $\hat{B}^{t\prime}(\pi^t) \leq 0$, then this is the unique equilibrium.*

6 Discussion and Conclusions

We have presented an overlapping generations model with PAYG pensions interacting with human capital accumulation and endogenous longevity. In this section we would like to discuss some interesting implications of our analysis and conclude.

First, note that if an economy is dynamically inefficient, then $\Delta^t(\pi^{t-1}) \geq 0$. Moreover, from the definition of $G(.)$ at the end of Section 3.1 we have that $G^t(wh^{t-1}, \pi^{t-1}) < 0$. Consequently, we have (recall condition (21)) that zero human capital accumulation is never an equilibrium. In this case, a well-known policy conclusion is that a PAYG pension system can be beneficial since it reduces the stock of capital. In our endogenous growth model, if the economy is dynamically inefficient, pensions are always compatible with growth.

Second, PAYG pensions do not necessarily harm growth in a dynamically efficient economy. In fact, in our setup PAYG pensions only need to be sufficiently small to ensure endogenous growth. Specifically, given inherited longevity, fertility and prices, π^{t-1}, n, R and w, the previous generation's pension rate must be sufficiently low so that $\pi^{t-1}p(\pi^{t-1})[\frac{n-R}{nR}] > G(w\kappa(\pi^{t-1}), \pi^{t-1})$ (recall discussion of (21)). Thus, sufficiently low pension rate $p(\pi^{t-1})$ ensures that $e^t > 0$ and hence $\pi^t > \pi^{t-1}$. Interestingly, if pensions are elastic and the initial pension rate is *sufficiently* low to ensure positive effort from generation $t = 1$, then we will have permanent endogenous growth. To see this, note that $\pi^0 p(\pi^0)[\frac{n-R}{nR}]$ $> G(w\kappa(\pi^0), \pi^0)$ ensures that $e^1 > 0$ and hence $\pi^1 > \pi^0$. Furthermore, if $\pi p(\pi)$ is strictly decreasing in π, we have, when $R > n$, that $\Delta^t(\pi^{t-1})$ is strictly increasing in π^{t-1}. Given that $G(w\kappa(\pi), \pi)$ is strictly decreasing in π, we therefore have that effort is positive in every period $t \geq 1$. Thus, since as shown by Abel et al. (1989), most countries are dynamically efficient, we have that PAYG pension rates need to be sufficiently low and pensions need to be elastic. This provides, in other words, another reason for decreasing pensions in line with the increase in life expectancy.

Furthermore, in our set-up moving to a fully-funded pension scheme can take place while compensating the initial losers. To see this, note that by setting $p^t = 0$ for any $t \geq 0$ we have that $G^t = 0$ and, given our assumptions so far, that $e^t > 0$ for any generation $t \geq 1$. That is, the economy will be growing permanently from period $t = 1$ and onwards. However, by setting $p^t = 0$, $t \geq 0$, and hence abolishing PAYG pensions from period $t = 2$ and onwards, generation $t = 0$ loses. The reason is that they will not receive any PAYG pension in seniority, despite the fact that they will contribute for the pensions of the old citizens in period $t = 1$. Nevertheless, this generation can be compensated by exploiting the permanent growth of the economy. In more detail, the government can borrow in period $t = 2$ from members of generation $t = 1$ to compensate the members of generation $t = 0$ for their foregone PAYG pensions. Specifically, the government will have to borrow from each adult of period $t = 2$ the amount of $\frac{\pi^0 p(\pi^0)}{n(1+\mu(e^1))} \equiv d_2$ per unit of labour income of the adults of period $t = 2$. This in turn will require borrowing from each adult of period $t = 3$ the amount of $(\frac{\pi^0 p(\pi^0)}{n(1+\mu(e^1))})\frac{R}{n(1+\mu(e^2))} \equiv d_3$

per unit of labour income of the adults of period $t = 3$, and so on. Under this scheme, adults of each generation $t \geq 1$ will simply adjust their net savings so that at optimum they still save at a gross rate s^t of their labour income. However, this will not affect growth in our economy. The reason is twofold. On the one hand, in our economy the engine of endogenous growth is human capital accumulation. On the other hand, $G^t = 0$ and hence $e^t > 0$ for any $t \geq 1$. Moreover, the period-1 present value of period-t public debt as a proportion of the total labour income in period t, $t \geq 2$, required under this scheme, is $\frac{d_t}{R^{t-1}} \equiv (\frac{\pi^0 p(\pi^0)}{R}) \frac{1}{\prod_{j=1}^{t-1}(n)^j(1+\mu(e^j))}$, where we use the convention that $\prod_{i=1}^{0} x^i = 1$. Clearly, then, $\lim_{t\to\infty} \frac{d_t}{R^{t-1}} = 0$, due to $n \geq 1$ and $e^{j+1} > 0$. Hence, in our setup, it is possible to build a Pareto-improving transition to a fully funded system in a dynamically efficient economy. This result is common with other papers on endogenous growth with positive externalities (see for example Belan et al., 1998). However, we derive it in a different framework where compensation to the last adult generation, that pay taxes but will not receive a pension, is made possible by an unbounded human capital driven growth once the economy has escaped from the zero growth equilibrium.

In conclusion we see this paper as a first attempt to study the interactions of pension programs with endogenous longevity in an endogenous growth model with human capital accumulation. In this setting, the inclusion of endogenous fertility and of the role of public and private health expenditure on longevity and their interactions with the pension policy deserves future study.

References

[1] Abel, A.B., Mankiw, N.G. and L.H. Summers, 1989, "Assessing Dynamic Efficiency: Theory and Evidence", *Review of Economic Studies*, 1989, *56*, 1-19.

[2] Aldersen, P. and D.P. Broer, 2005, "Ageing, Fertility and Growth", *Journal of Public Economics,* 89, 1075-1095.

[3] Anders, T.M., 2008, "Increasing Longevity and Social Security Reforms - A Legislative Procedure Approach", *Journal of Public Economics,* 92, 633-646.

[4] Belan, P., Michel P. and P. Pestieau, 1998, "Pareto-Improving Social Security Reform", *The Geneva Papers on Risk and Insurance Theory*, 23, 119-125.

[5] Blackburn, K. and G.P. Cipriani, 2002, "A Model of Longevity, Fertility and Growth", *Journal of Economic Dynamics and Control*, *26*, 187-204.

[6] Barro, R. J. and X. Sala-i-Martin, 2004, Economic Growth, MIT Press.

[7] Cervellati, M. and U. Sunde, 2005, "Human Capital Formation, Life Expectancy, and the Process of Development", *American Economic Review,* 95, 1653-1672.

[8] Cigno, A., 1992, "Children and Pensions", *Journal of Population Economics,* 5, 175-184.

[9] Cipriani, G.P. and M. Makris, 2007, "Indeterminacy, Intergenerational Redistribution, Endogenous Longevity and Human Capital Accumulation", *Journal of Economic Dynamics and Control*, 31, 613-633.

[10] Cremer, H., F. Gahvari and P. Pestieau, 2006, "Pensions with Endogenous and Stochastic Fertility", *Journal of Public Economics,* 90, 2303-2321.

[11] Cutler, D., A. Deaton and A. Llears-Muney, 2006, "The Determinants of Mortality", *Journal of Economic Perspectives,* 20, 97-120.

[12] Davis, J.B. and P. Kuhn, 1992, "Social Security, Longevity and Moral Hazard", *Journal of Public Economics*, 49, 91-106.

[13] Docquier, F., O. Paddison and P. Pestieau, 2007, "Optimal Accumulation in an Endogenous Growth Setting with Human Capital", *Journal of Economic Theory*, 134, 361-378.

[14] Echevarria C.A. and A. Iza, 2006, "Life Expectancy, Human Capital, Social Security and Growth", *Journal of Public Economics*, 90, 2323-2349.

[15] Ehrlich, I. and F. Lui, 1998, "Social Security, the Family and Economic Growth", *Economic Inquiry*, 36, 390-409.

[16] Ehrlich, I. and Kim, J., 2005, "Social Security, Demographic Trends, and Economic Growth: Theory and Evidence from the International Experience", *NBER Working Paper Series*, 11121.

[17] Furnée, C.A., Groot, W. and H. Maassen van den Brink, 2008, "The Health Effects of Education: A Meta-Analysis", *European Journal of Public Health*, forthcoming.

[18] Lagerlöf, N.P., 2003, "From Malthus to Modern Growth: Can Epidemics Explain the Three Regimes?", *International Economic Review*, 44, 755-778.

[19] Nishimura, K. and J. Zhang, 1992, "Pay-As-You-Go Public Pensions with Endogenous Fertility", *Journal of Public Economics*, 48, 239-258.

[20] Pestieau, P., Ponthiere, G. and M. Sato, 2006, "Longevity and Pay-As-You-Go Pensions", *mimeo.*

[21] Philipson T.J. and G.S. Becker, 1998, "Old-Age Longevity and Mortality-Contingent Claims", *Journal of Political Economy*, 106, 551-573.

[22] Poutvaara, P., 2007, "Social Security Incentives, Human Capital Investment and Mobility of Labor", *Journal of Public Economics*, 91, 1299-1325.

[23] Rosati, F.C., "Social Security in a Non-Altruistic Model with Uncertainty and Endogenous Fertility", *Journal of Public Economics,* 1996, 60, 283-294.

[24] Sinn, H.W., 2004, "The Pay-As-You-Go Pension System as a Fertility Insurance and an Enforcement Device", *Journal of Public Economics*, 88, 1335-1357.

[25] Tamura, R., 2006, "Human Capital and Economic Develpment", *Journal of Development Economics,* 79, 26-72.

[26] Wheeler, C.H., 2008, "Human Capital Externalities and Adult Mortality in the U.S.", *Federal Reserve Bank of St. Louis Working Paper,* 2007-045C.

[27] Wigger, B.U., 1999, "Pay-As-You-Go Financed Public Pensions in a Model of Endogenous Growth and Fertility", *Journal of Population Economics*, 12, 625-640.

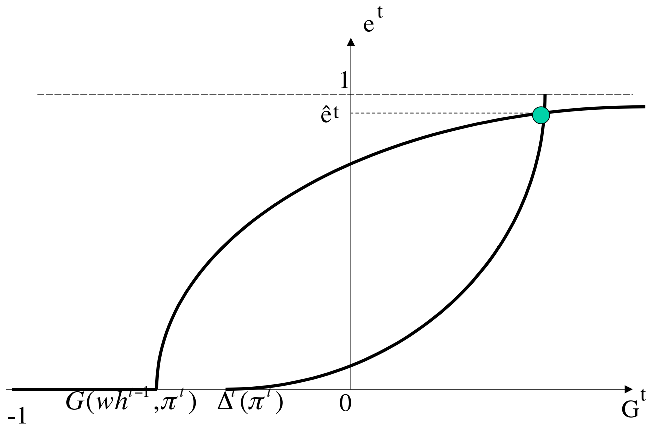

Fig. 1 Positive Effort under Exogenous Longevity

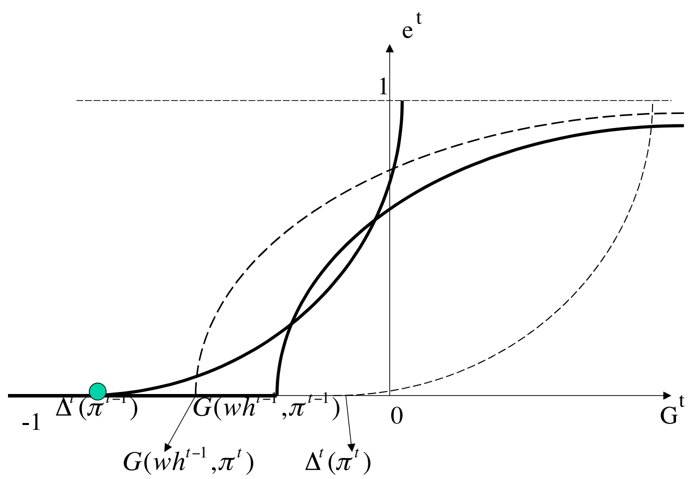

Fig. 2 Zero Effort under Endogenous Longevity