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Working Papers

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ChilD n. 21/2008

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Parameters Heterogeneity in a Model of Labour Supply: Exploring the Performance of Mixed Logit

Ugo Colombino* and Marilena Locatelli**

Abstract: In this note we investigate the empirical differences between the Random Utility model with fixed coefficients (Conditional Logit), and the Random Utility model with random coefficients (Mixed Logit). We consider a model of household labour supply developed for a project aimed at the evaluation of alternative Basic Income mechanisms. Data are drawn from the 1998 Bank of Italy survey of household income and wealth (SHIW 1998) and choice alternatives are generated using EUROMOD. We compare the estimates of the Conditional Logit and Mixed Logit. We also compare the respective results from simulating the effects of a Flat Tax reform. Although on average the estimates of Conditional Logit and of Mixed Logit are very close, the Mixed Logit estimates reveal that there is a significant unobserved heterogeneity of preferences. We also compare the simulations of a hypothetical Flat Tax reform. Although the differences are small, yet the results would imply different policy conclusions depending on whether Conditional Logit or Mixed Logit is adopted.

Keywords: labour supply, conditional logit, mixed logit, unobserved heterogeneity

JEL classifications: D0, J0

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Acknowledgements

This note results from research work conducted for two related CHILD projects directed by Ugo Colombino: (1) Guaranteed Minimum Income. A Crucial Node in the Design of Social Policy in Europe, financed by the Compagnia di San Paolo; (2) Modelli microeconometrici per la valutazione dei programme di Reddito Minimo Garantito, financed by the Ministry of University and Research (project 2006138400_002) and by the Compagnia di San Paolo.

The authors are thankful for the precious suggestions provided by Arne Risa Hole in interpreting the STATA code for Mixed Logit estimation.

1. Introduction

The adoption of the Random Utility approach in the modelling of labour supply represents an important innovation (Van Soest, 1995; Aaberge, Dagsvik and Strøm, 1995).

The basic model has then been generalized with the introduction of random choice sets (e.g. Aaberge, Dagsvik and Strøm, 1995; Aaberge, Colombino and Strøm, 1999; Dagsvik and Strøm, 2006). In those empirical papers the estimations were mostly carried out resorting to Conditional Logit (McFadden 1974).

A further generalization can be obtained by allowing for random coefficients. McFadden and Train (2000) propose the so-called Mixed Logit: a flexible model that can approximate any Random Utility model and overcome some of the limitations of Conditional Logit by allowing for random taste variation (random coefficients), unrestricted substitution patterns, and correlation in unobserved factor over time (Train 1998, Train and Revelt 1998, Train and Huber 2001, Train 2003, Hensher and Greene, 2003, Haan and Uhlendorff 2006).

The purpose of this paper is to explore the implications of Mixed Logit in modelling labour supply.

Section 2 presents the specification of Mixed Logit. Section 3 illustrates the data. Section 4 deals with the results of the estimates and of the simulations. Section 5 concludes with final remarks

2. The model

This note implements different empirical choice models to examine the labour supply choice: a conditional logit (McFadden 1974) and mixed logit choice models (Train 2003, Hensher and Greene 2003, Haan and Uhlendorff 2006, Colombino and Nese 2008), with two different assumptions on the random coefficients: i) normally distributed, ii) correlated and normally distributed.

In Subsection 2.1 we present a briefly description of the conditional and the mixed logit models and afterwards in the specific Subsections 2.2÷2.4 we present the details of the conditional logit model specifically referred to labour supply.

2.1 Conditional Logit and Mixed Logit

In this section we consider for simplicity a linear-in-parameters utility. The generalization to the non linear case is straightforward. When adopting the Conditional Logit model (McFadden 1974), we define the utility attained by individual *n* when choosing alternative *j* among *J* alternatives as follows:

(1) $$U_{nj} = \beta' x_{nj} + \varepsilon_{nj}$$

where x_{nj} is the vector of the observed explanatory variables and includes attributes of the alternatives, and socio-economic characteristics of the individual, β is a vector of (fixed) parameters to be estimated and ε_{nj} is a random variable that measures unobserved components of utility. We impose the condition that the ε_{nj} are i.i.d. type I extreme value. The decision maker chooses alternative *i* if and only if $U_{ni} > U_{nj}$, with $j \neq i$. The probability that individual *n* chooses alternative *i* turns out to be:

(2) $$P_{nj} = \frac{e^{\beta' x_{nj}}}{\sum_{r=1}^{J} e^{\beta' x_{nr}}}$$

In the first Mixed Logit model (normally distributed random coefficients), we drop the assumption of fixed taste parameters to take into account unobserved heterogeneity.

The utility of individual *n* choosing alternative *j* is now specified as:

(3) $$U_{nj} = \beta_n' x_{nj} + \varepsilon_{nj}$$

where:

β_n is a (individual-specific) vector of parameters that represents the marginal utilities of the characteristic x_{nj} for individual *n*;

The coefficient vector β_n is unobserved for each *n* and varies in the population with density $f(\beta \mid \theta^*)$ where θ^* are the *true* parameters of this distribution. Therefore, in this model the researcher does not estimate the coefficient β, but the parameter θ of the density function $f(\beta)$ of β (e.g. the mean and the variance-covariance matrix of β).

The choice probability for the situation j is obtained by computing the expectation of (2) with respect to $f(\beta)$

(4) $$P_{nj} = \int \left(\frac{e^{(\beta' x_{nj})}}{\sum_{r=1}^{J} e^{(\beta' x_{nr})}} \right) f(\beta)\, d\beta$$

The integral appearing in expression (4) usually cannot be evaluated in closed form, and simulation methods are used instead. In practice, M values $\beta^1, \beta^2, ..., \beta^M$ of β are sampled from a chosen distribution $f(\beta)$ and, on the bases of these values, the simulated probability is computed:

(5) $$\hat{P}_{nj} = \frac{1}{M} \sum_{m=1}^{M} \frac{e^{(\beta^{m'} x_{nj})}}{\sum_{r=1}^{J} e^{(\beta^{m'} x_{nr})}}$$

The simulated probabilities are inserted into the likelihood function to obtain a simulated likelihood. The distribution parameter $f(\beta)$ are therefore obtained by maximizing the simulated likelihood.

In the second Mixed Logit model, we assume that the random coefficients are correlated and normally distributed. The covariance matrix for the random coefficients is given by

(6) $$M = LL'$$

where the L matrix is the Cholesky factorization of the covariance matrix M.

2.2 The model of labour supply

Following Colombino at al., 2008, we will consider households with two decision-makers (i.e. the two partners). Of course, there might be other people in the household, but their behaviour is considered as exogenous.

Household n is assumed to maximize a utility function

$$(7) \qquad U_n(C_n, h_F, h_M)$$

under the constraints

$$h_F \in \Omega$$
$$h_M \in \Omega$$
$$C_n = R(w_{n,F}h_{n,F}, w_{n,M}h_{n,M}, y_n)$$

where

h_g = average weekly hours of work required by the j-th job in the choice set for partner g , $g = F$ (female) or M (male);

Ω = set of 12 discrete values (see Subsection 2.4 below);

$w_{n,g}$ = hourly wage rate of partner g;

y_n = vector of exogenous household gross incomes;

C_n = net disposable household income;

R = tax-transfer rule that transforms gross income into net disposable household income.[1]

The first two constraints say that the hours of work h_g are chosen within a discrete set of values Ω, including also 0 hours (i.e. non-participation). This discrete set of values of hours of work can be interpreted as the actual choice set (may be determined by institutional constraints) or as approximations to the true (possibly continuous) choice set.

[1] The tax-transfer rule is applied to yearly incomes, which are obtained by multiplying the average weekly incomes by 52.

The third constraint says that net income C_n is the result of a tax-transfer rule *R* applied to gross incomes.

The utility function is written as the sum of a systematic part and a random component:

$$(8) U_n(C_n, h_{n,F}, h_{n,M}) = V(C_n, h_{n,F}, h_{n,M}; Z_n, \vartheta) + \varepsilon = V(R(w_{n,F}h_{n,F}, w_{n,M}h_{n,M}, y_n), h_{n,F}, h_{n,M}; Z_n, \vartheta) + \varepsilon$$

where Z_n is a vector of household characteristics, ϑ is a vector of parameters to be estimated and ε is a random variable capturing the effect of unobserved (by the econometrician) variables upon the evaluation of U_n.

Under the assumption that ε is i.i.d. extreme value, it is well known[2] that the probability that a given household chooses $h_F = f, h_M = m$ is given by

$$(9) \qquad P_n(f, m; \vartheta) = \frac{\exp\left\{V\left(R(w_{n,F}f, w_{n,M}m, y_n), f, m; Z_n, \vartheta\right)\right\}}{\sum_{h_F \in \Omega}\sum_{h_M \in \Omega} \exp\left\{V\left(R(w_{n,F}h_F, w_{n,M}h_M, y_n), h_F, h_M; Z_n, \vartheta\right)\right\}}$$

2.3 Empirical specification of preferences

We choose a linear-in-parameters quadratic specification since it represents a good compromise between flexibility and ease of estimation. Hence, for each household, we write:

$$(10) \qquad \begin{aligned} V = {} & \theta_C C + \theta_F(T - h_F) + \theta_M(T - h_M) + \\ & + \theta_{CC}C^2 + \theta_{FF}(T - h_F)^2 + \theta_{MM}(T - h_M)^2 + \\ & + \theta_{CF}C(T - h_M) + \theta_{CM}C(T - h_M) + \theta_{FM}(T - h_F)(T - h_M) \end{aligned}$$

where T denotes total available time.

Some of the above parameters θs are made dependent on household or individual characteristics:

[2] See for example Ben -Akiva and Lerman (1985).

(11)

$$\theta_F = \beta_{F0} + \beta_{F1}\left(\text{Age of the wife}\right) + \beta_{F2}\left(\#\text{Children}\right) + \beta_{F3}\left(\#\text{Children under 6}\right) + \beta_{F4}(\#\text{Children 6-10})$$
$$\theta_M = \beta_{M0} + \beta_{M1}\left(\text{Age of the husband}\right) + \beta_{M2}\left(\#\text{Children}\right) + \beta_{M3}\left(\#\text{Children under 6}\right) + \beta_{M4}(\#\text{Children 6-10})$$
$$\theta_C = \beta_{C0} + \beta_{C1}\left(\#\text{Children}\right) + \beta_{C2}\left(\#\text{Children under 6}\right) + \beta_{C3}(\#\text{Children 6-10}).$$

2.4 Empirical specification of the opportunity sets

We assume that each partner can choose between 10 equally spaced values (from 1 to 80, each 8 hours wide[3]) of weekly hours of work. Moreover they can also choose to be out-of-work, either as non-participants or as unemployed (looking for a job) (12 choices each partner). Therefore, each household chooses among 144 alternatives. In order to compute net household income C for each one of the household jobs contained in $\Omega \times \Omega$, we use the EUROMOD Microsimulation model.[4] In other words, EUROMOD mimics the tax-transfer rule R. wage rates for those who are observed as not employed are imputed based on a wage equation estimated on the employed sub sample and corrected for sample selection.[5]

Most countries show a more or less pronounced concentration of people around hours corresponding to full-time, part-time and non-working. The model outlined above is typically unable to reproduce these peaks. A useful trick consists in adding dummies. We define the following dummies for part-time, full-time, overtime, non-working and non-working but looking for work respectively

[3] In detail the values are the following: 1,8, 9-16, 17-24, 25-32, 33-40, 41-48, 49-56, 57-64, 65-72, 73-80

[4] An overview of the EUROMOD project is provided by Bourguignon et al. (2000).

[5] The wage equations are available from the authors upon request.

(12)

$$D_{g1}\left(h_g\right)=\begin{cases}1 \text{ if } 17\le h_g \le 32\\ 0 \text{ otherwise}\end{cases}$$

$$D_{g2}\left(h_g\right)=\begin{cases}1 \text{ if } 33\le h_g \le 48\\ 0 \text{ otherwise}\end{cases}$$

$$D_{g3}\left(h_g\right)=\begin{cases}1 \text{ if } 49\le h_g\\ 0 \text{ otherwise}\end{cases}$$

$$D_{g4}\left(h_g\right)=\begin{cases}1 \text{ if } 0< h_g\\ 0 \text{ otherwise}\end{cases}$$

$$D_{g5}\left(h_g\right)=\begin{cases}1 \text{ if } h_g = 0 \text{ and looking for work}\\ 0 \text{ otherwise}\end{cases}$$

for $g = F$ (female) or M (male).

It can be shown that the dummies can be interpreted as reflecting quantity constraints on the labour market and different availability of opportunities (as in Aaberge et al., 1995, 1999), or specific utility of different types of jobs (as in van Soest, 1995), or both.

We then rewrite the choice probabilities as follows:

(13)

$$P_n(f,m;\vartheta)=\frac{\exp\left\{V\left(R(w_{n,F}f,w_{n,M}m,y_n),f,m;Z_n,\theta\right)+\sum_{k=1}^{5}\gamma_{Fk}D_{Fk}\left(f\right)+\sum_{k=1}^{5}\gamma_{Mk}D_{Mk}\left(m\right)\right\}}{\sum_{h_F\in\Omega}\sum_{h_M\in\Omega}\exp\left\{V\left(R(w_{n,F}h_F,w_{n,M}h_M,y_n),h_F,h_M;Z_n,\theta\right)+\sum_{k=1}^{5}\gamma_{Fk}D_{Fk}\left(h_F\right)+\sum_{k=1}^{5}\gamma_{Mk}D_{Mk}\left(h_M\right)\right\}}$$

where the γs are parameters to be estimated and where Z_n denotes the vector of characteristics (Age of the partners, Number and Age of the children) of household n.

Expression (13) corresponds to the Conditional Logit formulation, where the parameters ϑ are assumed to be fixed and common to all the households in the population.

When adopting the Mixed Logit we assume instead that the parameters ϑ (subset of them) are random with density $\varphi(\vartheta;\mu,\Sigma)$ defined up to a mean vector μ and a covariance matrix Σ. Then t·he appropriate expression for the choice probabilities becomes:

(14) $$P_n(f,m;\mu,\Sigma)=\int P_n(f,m;\vartheta)\varphi(\vartheta;\mu,\Sigma)d\vartheta$$

where $P_n(f,m;\vartheta)$ is defined as in expression (13). The choice probability can be easily approximated with a simulation technique as exemplified by expression (5).

If (f_n, m_n) is the observed choice for the n-th household, the ML estimates are

(15) $$\vartheta^{ML} = \arg\max_{\vartheta} \sum_{n=1}^{N} \ln P_n(f_n, m_n; \vartheta)$$

for Conditional Logit

and

(16) $$\left(\mu^{ML}, \Sigma^{ML}\right) = \arg\max_{\vartheta} \sum_{n=1}^{N} \ln P_n(f_n, m_n; \mu, \Sigma)$$

for Mixed Logit.

Once the parameters of the model are estimated, we are interested in simulating various results under alternative tax-transfer rules. Suppose we want to simulate the expected value of some function $q^n(f,m)$: it might be the net available income under the new rule, or hours worked etc. Then we compute the expected value of as

(17) $$E(q^n(f,m)) = \sum_{f\in\Omega}\sum_{m\in\Omega} q^n(f,m)P^n(f,m;\vartheta^{ML})$$

in the Conditional Logit case, and as

(18) $$E(q^n(f,m)) = \int \sum_{f\in\Omega}\sum_{m\in\Omega} q^n(f,m)P^n(f,m;\vartheta)\varphi(\vartheta;\mu^{ML},\Sigma^{ML})d\vartheta$$

in the Mixed Logit case.

One of the criteria we use in order to evaluate and compare different tax-benefit rule is social welfare. It is computed as a Social Welfare function that takes as arguments the individual welfare level attained by the households under the tax-transfer rule. Let S be the average (across households) of individual welfare and *I* be the Gini-index of the distribution of individual welfare. Then social welfare is measured by S(1 – *I*).[6]

[6] This form is known in the literature as the Sen's Social Welfare Function. It can also be shown that tit is a member of the class of rank-dependent social welfare functions (see Aaberge, 2007).

We present two versions. The first one uses the expected maximum utility attained by the household as the measure of individual welfare, i.e.[7]

(19) $$E \max U^n = \ln\left(\sum_{h_F \in \Omega}\sum_{h_M \in \Omega} \exp\left\{V\left(R(w_F^n h_F, w_M^n h_M, y^n), h_F, h_M; \overline{Z}, \theta\right) + \sum_{k=1}^{5}\gamma_{Fk}D_{Fk} + \sum_{k=1}^{5}\gamma_{Mk}D_{Mk}\right\}\right)$$

in the Conditional Logit case, and

(20) $$E \max U^n = \int \ln\left(\sum_{h_F \in \Omega}\sum_{h_M \in \Omega} \exp\left\{V\left(R(w_F^n h_F, w_M^n h_M, y^n), h_F, h_M; \overline{Z}, \theta\right) + \sum_{k=1}^{5}\gamma_{Fk}D_{Fk} + \sum_{k=1}^{5}\gamma_{Mk}D_{Mk}\right\}\right)\varphi(\vartheta; \mu^{ML}, \Sigma^{ML})d\vartheta$$

in the Mixed Logit case, where $\overline{Z}$ is the vector of the sample average of the household characteristics. We use a common value of characteristics in order to insure comparability of individual welfare measures.[8] The second version more simply adopts the expected attained net available income (computed according to expressions (17) or (18)) as a measure of individual household welfare.

3. The data

The data used in this study are selected from the Bank of Italy survey of households' income and wealth (SHIW, 1998) and cover married as well as cohabiting couples that are employed, unemployed, or inactive. The age of the partners ranges from 20 to 55. None of them are self-employed or on disability or on other type of benefits, none is civil servant, none disabled, and none student.

The original data set includes data on the actual choice. A new data set has been generated including potential choices[9] using EUROMOD tax-benefit model that allows simulating the disposable income associated with each choice[10].

Table 1 provides the descriptive statistics of the observed variables in the original dataset, while Table 2 reports the description of the variables used in the estimations.

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[7] For the derivation of this expression, see Ben-Akiva and Lerman (1985). This same methodology for empirical welfare evaluation is used by Colombino (1998).

[8] For the foundations of this procedure see for example Deaton and Muellbauer (1980).

[9] The dataset was generated by Cathal O'Donoghue for a previous paper (Colombino U., et al., 2008), in a form appropriate for the estimation of the micro econometric model starting from the original sources.

[10] An overview of the EUROMOD project is provided by Bourguignon et al. (2000).

Table 1. Descriptive statistics of the observed variables - number of observations 2326 – Values in Euro

Variable	Mean	Std.Dev.	Min	Max
Hours_male	34.71	15.99	0	150
Hours_female	15.44	17.92	0	90
Age_male	43.11	7.71	20	55
Age_female	39.77	7.76	20	55
Children aged 0-5	0.30	0.56	0	3
Children aged 6-10	0.33	0.56	0	3
Gross income	2254.99	1636.80	-5387.77	15815.86
Net income	1812.77	1072.97	-5639.13	8663.37
Taxes	514.16	473.48	0	5629.97
Benefits	166.48	409.35	0	4434.81
Social insurance	94.55	117.87	0	1522.52
Hourly wage_male	10.52	7.5	90	147.86
Hourly wage_female	7.32	4.56	0	81.04
Participation_male	0.86	0.35	0	1
Participation_female	0.46	0.50	0	1
Unemployment_male	0.06	0.23	0	1
Unemployment_female	0.09	0.28	0	1

Table 2. Description of the variables of the new data set used in the estimation

Variable	Description
dispy	disposable income
dispyquare	disposable income squared
Wm	dummy: 1 if woman working hours > 0, 0 otherwise
Wf	dummy: 1 if man working hours > 0, 0 otherwise
Pm	dummy: 1 if man working part time (weekly hours >17 and <32), 0 otherwise
Pf	dummy: 1 if woman working part time (weekly hours >17 and <32), 0 otherwise
Gm	dummy: 1 if man are working full time (weekly hours > 32 and ≤48), 0 otherwise
Gf	dummy: 1 if woman are working full time (weekly hours > 32 and ≤48), 0 otherwise
Om	dummy: 1 if man are working over hours (weekly hours >48), 0 otherwise
Of	dummy: 1 if woman are working over hours (weekly hours >48), 0 otherwise
Um	dummy: 1 if woman are unemployed, 0 otherwise
Uf	dummy: 1 if man are unemployed, 0 otherwise
l_ff	hour woman leisure (Total weekly hours (100)- female hours of work)
l_mm	hour man leisure (Total weekly hours (100)- male hours of work)
l_f_square	woman leisure squared
l_m_square	man leisure squared
l_mx	interaction of of disposable income with man leasure
l_fx	interaction of disposable imcome with woman leasure
l_fm	interaction of man leasure with woman leasure
l_age_f	interaction of woman age with woman leasure
l_age_m	interaction of man age with man leasure
l_nch_m	interaction of number of children with man leasure
l_nch_f	interaction of number of children with woman leasure
l_nch05_m	interaction of number of children aged 0-5 with man leasure
l_nch05_f	interaction of number of children aged 0-5 with woman leasure
l_agex_m	interaction of man age with disposable income
l_agex_f	interaction of woman age with disposable income
l_nchx	interaction of number of children with disposable icome
l_nchx_05	interaction of number of children aged 0-5 with disposable income
l_nchx_610	interaction of number of children aged 6-10 with disposable income
l_nch610_m	interaction of number of children aged 6-10 with man leisure
l_nch610_f	interaction of number of children aged 6-10 with woman leisure

4. The results

This Section shows the results of the different estimation approaches of the model. Furthermore, we compare the results of the simulations of the current tax system and those of the flat tax system (both in the case with current benefit equal to zero, and in the case with current benefits included) whose tax rate was determined in such a way to generate the same total tax revenue as in 1998.

Table 3 provides the results related to the conditional logit model, while Table 4 provides the results of the mixed logit model.

In the mixed logit, estimated using STATA (Hole 2007), we assume that the coefficients for disposable income (*dispy*), man and woman hours of leisure (*l_mm* and *l_ff*), are normally distributed whereas the remaining coefficients are fixed. The model is estimated using 20 and 50 Halton draws, but the results do not change much in the two different simulations, so in Table 4 we report only the results of the model with 50 Halton draws. The estimated standard deviation of *dispy* is significant, indicating that parameters do indeed vary in the population, while the estimated standard deviations of the variables *l_ff* and *l_mm* are not significant. The likelihood ratio index[11] rises substantially from allowing the parameter to vary (0.0054 in the conditional logit and 0.5469 in the mixed logit). The magnitudes of the estimated standard deviations are reasonable with respect to the estimated mean: the distribution of *dispy* coefficient has an estimated mean of 0.002 and an estimated standard deviation of 0.0006.

Table 5 provides the results of the Mixed Logit with correlated normally distributed coefficients. In this model, the starting values are taken from the model with uncorrelated normal coefficients. The joint significance of the off-diagonal elements of the covariance matrix is tested using the likelihood test. The test statistic, which is chi-square distributed with 33 degrees of freedom under the null of uncorrelated coefficients, is given by 2*(7122.509-7074.549) = 95.92 implying that the null hypothesis is rejected.

Table 6 provides estimates - and standard errors of the estimates - of the terms of the covariance matrix of the random coefficients.

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[11] The likelihood ratio index is a measure of goodness of fit, defined as 1-[SLL(θ_e)/SLL(0)], where SLL(θ_e) is the value of the simulated log likelihood function at the estimated parameters and SLL(0) is the value with all parameters equal to zero.

Table 3. Conditional Logit estimates

	Coef.	Std. Err.	z
Wm	-3.660665	0.3076515	-11.90
Wf	-4.423441	0.2473933	-17.88
Pm	0.562867	0.2898244	1.94
Pf	1.412638	0.2511447	5.62
Gm	3.236227	0.354282	9.13
Gf	2.998544	0.3231668	9.28
Om	2.342209	0.3666446	6.39
Of	1.586432	0.4110833	3.86
Um	-0.1486253	0.1290116	-1.15
Uf	-1.792578	0.0775307	-23.12
disp	0.001364	0.0005326	2.56
dispysquare	-1.04E-07	2.30E-08	-4.51
l_ff	0.3316859	0.0499526	6.64
l_f_square	-0.0027379	0.0003576	-7.66
l_mm	0.1901558	0.036505	5.21
l_m_square	-0.0023331	0.0002726	-8.56
l_mx	0.0000187	2.26E-06	8.27
l_fx	-0.0000123	1.87E-06	-6.58
l_fm	0.0013018	0.0001218	10.69
l_age_f	0.0012231	0.0002865	4.27
l_nch_f	0.001047	0.0027956	0.37
l_nch05_f	0.0092827	0.004353	2.13
l_age_m	0.0001199	0.0003576	0.34
l_nch_m	-0.0050083	0.0034171	-1.47
l_nch05_m	0.0024943	0.0054501	0.46
l_agex_f	0.0000415	0.0000106	3.92
l_agex_m	-0.0000526	0.0000112	-4.7
l_nchx	-0.0001132	0.0000984	-1.15
l_nchx_05	0.0000461	0.0001561	0.29
l_nchx_610	0.0003463	0.0001662	2.08
l_nch610_f	0.0097335	0.0045826	2.12
l_nch610_m	0.0083289	0.0055396	1.5
N. Obs.			2324
LR Chi Square			8843.37
Prob > chi2			0
Pseudo R2			0.38
Log Likelihood at β= 0			-7166.59
Log Likelihood (at convergence)			-7128.16
Likelihood ratio index: 1-[-7128.163/-7166.591] =			0.0054

Note: the first 10 coefficients are the γs of expression (13); the other coefficients are the θs and βs of expressions (10) and (11).

Table 4. Mixed Logit estimates with independent normally distributed coefficients

	Coef.	Std. Err.	z
Wm	-3.616990	0.310405	-11.65
Wf	-4.424012	0.247445	-17.88
Pm	0.605451	0.291733	2.08
Pf	1.414666	0.251316	5.63
Gm	3.295563	0.357110	9.23
Gf	3.007177	0.323582	9.29
Om	2.406232	0.369520	6.51
Of	1.601941	0.411892	3.89
Um	-0.123519	0.130053	-0.95
Uf	-1.803158	0.077658	-23.22
dispysquare	0.000000	0.000000	-3.36
l_f_square	-0.002793	0.000360	-7.75
l_m_square	-0.002452	0.000343	-7.15
l_mx	0.000023	0.000003	6.89
l_fx	-0.000010	0.000002	-4.06
l_fm	0.001263	0.000134	9.41
l_age_f	0.001079	0.000318	3.39
l_nch_f	0.000208	0.003117	0.07
l_nch05_f	0.010058	0.004848	2.07
l_age_m	-0.000225	0.000423	-0.53
l_nch_m	-0.005902	0.003929	-1.50
l_nch05_m	0.003085	0.006230	0.50
l_agex_f	0.000046	0.000014	3.34
l_agex_m	-0.000072	0.000015	-4.77
l_nchx	-0.000160	0.000123	-1.31
l_nchx_05	0.000061	0.000194	0.32
l_nchx_610	0.000307	0.000198	1.55
l_nch610_f	0.008905	0.004979	1.79
l_nch610_m	0.007256	0.006242	1.16
disp ***(mean)***	0.001772	0.000657	2.70
l_ff ***(mean)***	0.349994	0.051234	6.83
l_mm ***(mean)***	0.223284	0.043566	5.13
disp ***(std.dev)***	0.000637	0.000157	4.06
l_ff ***(std.dev)***	0.001738	0.012108	0.14
l_mm ***(std.dev)***	0.018578	0.014554	1.28
N. Obs.			2324
LR Chi Square			11.31
Prob > chi2			0.01
Log Likelihood at $\beta = 0$			-15719.81
Log Likelihood (at convergence)			-7122.51
Likelihood ratio index:			
1-(-7122.509/-15719.814) =			0.55

Note: the first 10 coefficients are the γs of expression (13); the other coefficients are the θs and βs of expressions (10) and (11).

Table 5. Mixed Logit estimates with correlated normally distributed coefficients

	Coef.	Std. Err.	z
Wm	-3.870932	0.326953	-11.84
Wf	-4.404687	0.247965	-17.76
Pm	0.602692	0.295553	2.04
Pf	1.437634	0.253198	5.68
Gm	3.393109	0.363784	9.33
Gf	3.087686	0.326416	9.46
Om	2.578034	0.378576	6.81
Of	1.782573	0.415504	4.29
Um	-0.191925	0.131742	-1.46
Uf	-1.888456	0.079393	-23.79
dispysquare	-2.22E-07	4.85E-08	-4.58
l_f_square	-0.004670	0.000515	-9.07
l_m_square	-0.004365	0.000490	-8.90
l_mx	0.000047	5.02E-06	9.39
l_fx	-0.000030	5.30E-06	-5.67
l_fm	0.005295	0.000597	8.86
l_age_f	0.002108	0.000652	3.23
l_nch_f	0.004018	0.005595	0.72
l_nch05_f	0.019082	0.008977	2.13
l_age_m	-0.000682	0.000624	-1.09
l_nch_m	-0.008794	0.005671	-1.55
l_nch05_m	0.000882	0.009257	0.10
l_agex_f	0.000072	0.000018	3.97
l_agex_m	-0.000081	0.000016	-5.06
l_nchx	-0.000140	0.000156	-0.89
l_nchx_05	0.000261	0.000253	1.03
l_nchx_610	0.000482	0.000253	1.91
l_nch610_f	0.015513	0.008973	1.73
l_nch610_m	0.007486	0.008963	0.84
dispy ***(mean)***	0.002358	0.000904	2.61
l_ff ***(mean)***	0.418742	0.064451	6.50
l_mm ***(mean)***	0.125031	0.054794	2.28
dispy ***(std.dev)***	0.001574	0.000224	7.02
l_ff ***(std.dev)***	0.115374	0.014548	7.93
l_mm ***(std.dev)***	0.101742	0.013374	7.61
N. Obs.			2324
LR Chi Square			107.23
Prob > chi2			0.00
Log Likelihood at $\beta = 0$			-7122.51
Log Likelihood (at convergence)			-7074.55

Note: the first 10 coefficients are the γs of expression (13); the other coefficients are the θs and βs of expressions (10) and (11).

Table 6. Covariance matrix of the random coefficients

	Coef.	Std.Err.	z
v11	0.000002	0.000001	3.51
v21	0.000145	0.000036	4.05
v31	-0.000055	0.000026	-2.10
v22	0.013311	0.003357	3.97
v32	-0.009872	0.002603	-3.79
v33	0.010351	0.002721	3.80

Note: vij = covariance between variable i and variable j (variable 1 is the coefficient of dispy, variable 2 is the coefficient of l_ff, variable 3 is the coefficient of l_mm).

In order to exemplify the implications of adopting Conditional Logit or Mixed Logit, we performed a simulation of the effects on a hypothetical reform that replaces the current tax system with a flat tax system. We consider two versions: a flat tax without the current benefits (transfers) and a flat tax complemented by the current benefits. As in Colombino et al. (2008), in Tables 7-9 we show – for the current tax system, flat tax system without benefits and the flat tax system with current benefits – the following simulated variables: weekly working hours for men and women (hm and hf), disposable income (disp), Gini index of disposable income (G(C)) and of utility (G(U)), the tax rate (t), benefits (B), social welfare based on utility (S(U)) and social welfare based on income (S(C)). The simulations are performed under the constraint that the total net tax revenue be equal to the current one. The constraint is satisfied by iterating the simulation runs and updating the tax rate t.

When using Mixed Logit estimates we simulate the results of interest according to expressions (18) and (20). The integrals are approximated with the simulation technique exemplified in expression (5), with 30 values of the random parameters sampled from their estimated joint distribution.

Overall the results of Conditional Logit and Mixed Logit are rather close. Yet there are differences that in principle would lead to diverging policy conclusions. For example, the results based on Conditional Logit show that the reform Flat Tax (+ current benefits) is inferior to the current tax rule according to the S(U) criterion; on the other hand, the results based on Mixed Logit would judge the same reform superior to the current rule.

Table 7 Simulations using conditional and mixed logit estimation: Current tax system.

	Conditional Logit	**Mixed Logit**
Mean(U)	24.33	25.57
Gini(U)	0.02	0.03
Mean(C)	1816.044	1823.76
Gini (C)	0.17	0.23
hm	35.77	36.10
hf	14.34	14.21
Taxes	533.97	536.86
Gross	2260.60	2272.38
F	0.15	0.15
t	0.42	0.42
B	190.44	190.13
S(U)	23.78	24.91
S(C)	1513.64	1398.20

Table 8. **Simulations using conditional logit and mixed logit. Flat Tax without benefits.**

	Conditional Logit	**Mixed Logit**
Mean(U)	24.29	25.59
Gini(U)	0.03	0.03
Mean(C)	1895.49	1913.53
Gini (C)	0.28	0.27
hm	36.87	37.21
hf	15.36	15.48
Taxes	343.53	343.54
Gross	2343.72	2362.96
F	0.15	0.15
t	0.15	0.16
B	0.00	0.00
S(U)	23.65	24.90
S(C)	1373.28	1390.41

Table 9. Simulations using conditional and mixed logit estimation: Flat tax system with current benefits.

	Conditional Logit	Mixed Logit
Mean(U)	24.31	25.66
Gini(U)	0.03	0.03
Mean(C)	1805.24	1918.69
Gini (C)	0.26	0.26
hm	35.66	37.08
hf	14.35	15.46
Taxes	536.79	534.59
Gross	2248.71	2367.71
F	0.15	0.15
t	0.25	0.24
B	193.24	191.06
S(U)	23.65	24.95
S(C)	1331.63	1406.53

5. Concluding remarks

We have explored the empirical differences between the discrete choice model without random effect, namely the conditional logit, and with random effects, namely the mixed logit, using a household labour supply model.

Although the estimates do not show relevant differences in the (means of) estimated coefficients, the mixed logit approach reveals the presence of a significant unobserved heterogeneity effects in the preferences.

The simulation experiments suggest that the estimates based on Mixed Logit based on Conditional Logit on those might imply different policy recommendations.

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